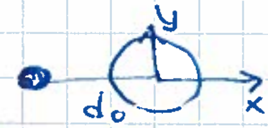
L07 Magnitude of tides

$m\vec{g} + \vec{F}_{tid}$ is normal to the ocean surface.

$m\vec{g} = -\nabla U \qquad F_{tid} = -\nabla U_{tid}$ - conservative F

$$U_{tid} = -GM_m m\left(\frac{1}{d} + \frac{x}{d_0^2}\right)$$



$\nabla(U + U_{tid})$ is normal to the surface

$U_{eff} = U + U_{tid}$ = const on the surface

$$U_{ef}(a) = U_{ef}(c) \to U(a) - U(c) = U_{tid}(c) - U_{tid}(a)$$

$U(a) - U(c) = mgh$ – h difference between high and low tides.

$$U_{tid}(c) = -GM_m m\frac{1}{d} = -GM_m m\frac{1}{d_0}\left(1 - \frac{1}{2}\frac{R^2}{d_0^2}\right), \quad d = \sqrt{d_0^2 + R^2}$$

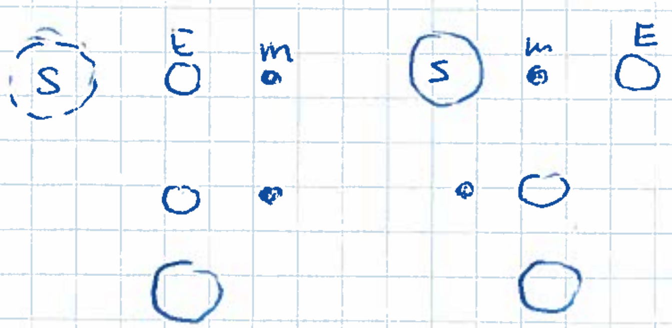
$$U_{tid}(a) = -GM_m m\left(\frac{1}{d} - \frac{R}{d^2}\right) \qquad d = d_0 - R$$

$$mgh = \frac{GM_m m}{d_0}\frac{3R^2}{2d_0^2} \to h = \frac{GM_m}{g d_0}\frac{3}{2}\frac{R^2}{d_0^2}$$

$R \approx 6400$ km. $d_0 \sim 380000$ km. $\frac{M_m}{M_E} = \frac{7\cdot 10^{22}}{6\cdot 10^{24}}$

$$h = \frac{3}{2}\frac{M_m}{M_E}\frac{R^4}{d_0^3} = R\cdot\frac{3}{2}\frac{M_m}{M_E}\frac{R^3}{d_0^3}$$

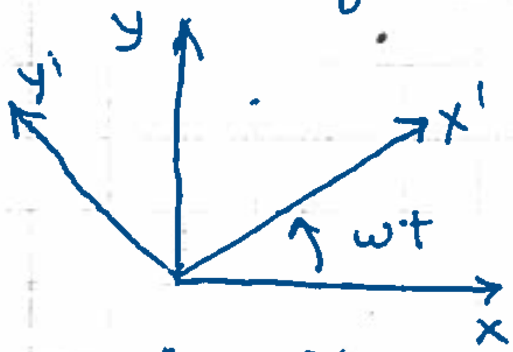
$h_{Em} \approx 54$ cm $\qquad h_{ES} \approx 25$ cm.



→ high (spring) tides

→ low (neap) tides

L07 * Rotating coordinates.



$T = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) \qquad U(x,y) = 0$

$x = x' \cos\omega t + y' \sin\omega t$

$y = y' \cos\omega t - x' \sin\omega t$

$\dot{x} = \dot{x}' \cos\omega t + \dot{y}' \sin\omega t - \omega x' \sin\omega t + y'\omega \cos\omega t =$

$= (\dot{x}' + y'\omega) \cos\omega t + (\dot{y}' - x'\omega) \sin\omega t$

$\dot{y} = (\dot{y}' - x'\omega) \cos\omega t - (\dot{x}' + y'\omega) \sin\omega t$

$\dot{x}^2 + \dot{y}^2 = (\dot{x}' + y'\omega)^2 + (\dot{y}' - x'\omega)^2$

$\mathcal{L} = \underbrace{\frac{1}{2}m(\dot{x}'^2 + \dot{y}'^2)}_{T} + \underbrace{\frac{1}{2}m\omega^2(x'^2 + y'^2)}_{-\,U_c} + \underbrace{m\omega(\dot{x}'y' - \dot{y}'x')}_{+\ \text{Coriolis term}}$

$U_c = \frac{1}{2}m\omega^2(x'^2 + y'^2)$ – "potential energy" due to centrifugal force

$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}'} = \frac{d}{dt}(m\dot{x}' + m\omega y') = m\omega^2 x' - m\omega \dot{y}'$

$m\ddot{x}' = m\omega^2 x' - 2m\omega \dot{y}'$