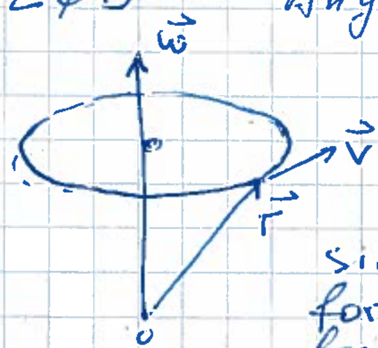
L08 Angular velocity



$$\vec{\omega} = |\omega| \hat{u} \Rightarrow \vec{V} = \vec{\omega} \times \vec{r}$$

or $\frac{d\vec{r}}{dt} = \dot{\vec{r}} = \vec{\omega} \times \vec{r}$

similar $\quad \dot{\vec{V}} = \vec{\omega} \times \vec{V} = \vec{\omega} \times \dot{\vec{r}}$

for any unit vector in the rotating frame: $\dot{\vec{e}} = \vec{\omega} \times \vec{e}$

- time derivative in rotating frame.

S_0 - inertial frame

S - non-inertial frame rotating with $\vec{\Omega}$ with respect to S_0 frame

for any vector $\vec{Q}$ $\left(\frac{d\vec{Q}}{dt}\right)_{S_0}$ - rate of change in S_0

relate it to $\left(\frac{d\vec{Q}}{dt}\right)_S$ - rate of change in S

$\vec{Q} = \sum_i Q_i \vec{e}_i \qquad \vec{e}_i$ - unit vectors in S

$$\left(\frac{d\vec{Q}}{dt}\right)_S = \dot{\vec{Q}}_S = \sum \frac{dQ_i}{dt} \vec{e}_i = \sum \dot{Q}_i \vec{e}_i$$

$$\dot{\vec{Q}}_{S_0} = \sum \dot{Q}_i \vec{e}_i + \sum Q_i \dot{\vec{e}}_i \qquad \dot{\vec{e}}_i = \vec{\Omega} \times \vec{e}_i$$

$$\dot{\vec{Q}}_{S_0} = \dot{\vec{Q}}_S + \sum Q_i \vec{\Omega} \times \vec{e}_i = \dot{\vec{Q}}_S + \vec{\Omega} \times \sum Q_i \vec{e}_i$$

$$\dot{\vec{Q}}_{S_0} = \dot{\vec{Q}}_S + \vec{\Omega} \times \vec{Q}$$

Newton's second law in rotating frame. page 342-343 textbook.

$m(\ddot{\vec{r}})_{S_0} = \vec{F} \qquad (\dot{\vec{r}})_{S_0} = (\dot{\vec{r}})_S + \vec{\Omega} \times \vec{r}$

$$(\ddot{\vec{r}})_{S_0} = \left(\frac{d}{dt}\right)_{S_0} \left[(\dot{\vec{r}})_S + \vec{\Omega} \times \vec{r}\right] = \left(\frac{d}{dt}\right)_S (\dot{\vec{r}}_S + \vec{\Omega} \times \vec{r}) + \vec{\Omega} \times (\dot{\vec{r}}_S + \vec{\Omega} \times \vec{r})$$

$$m\ddot{\vec{r}} = \vec{F} + 2m\dot{\vec{r}} \times \vec{\Omega} + m[\vec{\Omega} \times \vec{r}] \times \vec{\Omega}$$

Coriolis force $\qquad$ Centrifugal force

LØ8 Centrifugal Force / Coriolis Force.

inertial forces (fictitious)

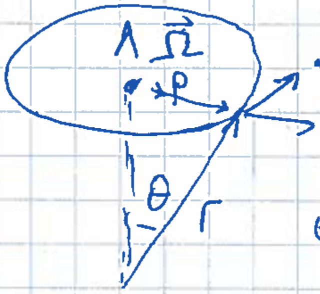
$F_{cor} \sim \vec{v} = \dot{\vec{r}}$ velocity in the rotating frame

$\vec{F}_{cor} \sim m v \Omega$ $\qquad F_{cf} \sim m r \Omega^2$ $\qquad \Omega \sim 7 \cdot 10^{-5} s^{-1}$

for rotating Earth: $R\Omega \sim 1600$ km/h $\qquad \frac{F_{cor}}{F_{cf}} \sim \frac{v}{R\Omega}$

Coriolis force becomes significant for $v \sim R\Omega$

Example: car $v \sim 100$ km/h $\qquad F_{cor}/F_{cf} \sim 0{,}07$



$\vec{v} = \vec{\Omega} \times \vec{r}$

$F_{cf} = m(\vec{\Omega} \times \vec{r}) \times \vec{\Omega}$

θ – latitude. $\qquad F_{cf} = m\Omega^2 r \sin\theta \; \hat{\rho}$

ρ – distance from axis

– Free Fall acceleration. $\qquad m\ddot{\vec{r}} = m\vec{g}_0 + \vec{F}_{cf} = m\vec{g}$

$$\vec{g} = \hat{g}_0 - \Omega^2 \sin\theta \; \hat{\rho} \; R$$

$g_{rad} = g_0 - \Omega^2 \sin^2\theta \; R$

$g_{tang} = \Omega^2 \sin\theta \cos\theta \, R$

at $\theta > 0$

g is not pointing to the center of the Earth

– $F_{cor} = 2m \dot{\vec{r}} \times \vec{\Omega} = 2m\vec{V} \times \vec{\Omega}$

V – object velocity in rotating frame

baseball: $V \sim 50$ m/s $\qquad \bar{a} = 2\vec{V} \times \Omega \sim 7 \cdot 10^{-3}$ m/s²

example 1

object deflected to the right

deflected to the left.

affects long range shooting

Example 2 hurricanes, typhoons $\}$ cyclons

creates circular motion of air