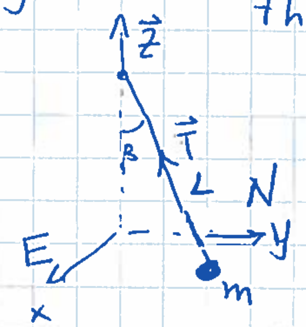
L09 **The Foucault Pendulum (1851)**
(Leon)



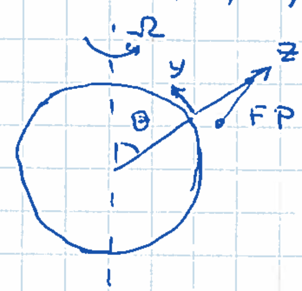
$$m\ddot{\vec r} = \vec T + m\vec g_0 + 2m\vec v\times\vec\Omega + m(\vec\Omega\times\vec r)\times\vec\Omega$$

$$m\ddot{\vec r} = \vec T + m\vec g + 2m\vec v\times\vec\Omega$$

$\hat g = \hat z$ $\qquad T_z = T\cos\beta \sim T \approx mg.$

$T_x = -mg\, x/L \qquad T_y = -mg\, y/L \qquad \vec\Omega = (0, \Omega\sin\theta, \Omega\cos\theta)$

$$\dot{\vec r}\times\vec\Omega = \begin{bmatrix} \hat x & \hat y & \hat z \\ \dot x & \dot y & \dot z \\ 0 & \Omega_y & \Omega_z \end{bmatrix} \qquad \dot{\vec r} = (\dot x, \dot y, \dot z)$$



Eq.1 $\quad \ddot x = -gx/L + 2\dot y\,\Omega_z$

$\qquad\quad \ddot y = -gy/L - 2\dot x\,\Omega_z$

$$\dot{\vec r}\times\vec\Omega = [\dot y\Omega_z - \overset{\nearrow 0}{\dot z\Omega_y},\ -\dot x\Omega_z,\ \dot x\Omega_y]$$

Eq.1 a) $\ddot x - 2\Omega_z\dot y + \omega_0^2 x = 0 \qquad \omega_0^2 = g/L$

b) $\ddot y + 2\Omega_z\dot x + \omega_0^2 y = 0 \qquad$ use $\eta = x + iy$

$$a+ib = \ddot\eta + 2i^2\Omega_z\dot y + 2i\Omega_z\dot x + \omega_0^2\eta = \ddot\eta + 2i\Omega_z\dot\eta + \omega_0^2\eta = 0$$

second order, linear, homogenious equation
solution in the form $\eta \sim e^{-i\alpha t}$

equation for α: $\quad \alpha^2 - 2\Omega_z\alpha - \omega_0^2 = 0$

$$\alpha = \Omega_z \pm \sqrt{\Omega_z^2 + \omega_0^2} \approx \Omega_z \pm \omega_0$$

$$\eta = e^{-i\Omega_z t}\left(C_1 e^{i\omega_0 t} + C_2 e^{-i\omega_0 t}\right)$$

use initial conditions: $x(0) = A,\ y(0) = 0,\ V_x = V_y = 0$

$C_1 = C_2 = A/2 \qquad \eta(t) = A e^{-i\Omega_z t}\cos\omega_0 t \qquad \omega_0 \gg \Omega_z$

The Foucault Pendulum.

$$\eta(t) = x(t) + iy(t) = Ae^{-i\Omega_z t}\cos\omega_0 t$$

$\omega_0 >> \Omega$: First FP: $L = 67m$ $\omega_0 = 0.38$; $\tau = 2.5$sec.

$\omega_0 \approx 0.38 >> 7.3 \cdot 10^{-5}$

latitude

$\Omega_z = \Omega\cos\theta = \Omega\sin\varphi$ NH: $\Omega_z > 0$ rotates clockwise
SH: $\Omega_z < 0$ rotate counterclock.

- poles: $\Omega_z = \Omega$ $(\theta = 0)$ full rotation a day.
- equator $\Omega_z = 0$ $(\theta = 90^\circ)$ fixed.
- $\varphi = 30^\circ$ $\Omega_z = 360^\circ \sin\varphi/\text{day} \Rightarrow 360^\circ$ in 2 days

Gainesville