L19 Several dimensions.

$\vec{q} = (q_1, q_2, \dots q_N)$ $\vec{P} = (P_1, P_2, \dots P_N)$

generalized coordinates conjugate momentum

if we use natural coordinates $\vec{q}$ so $\vec{r}_\alpha = \vec{r}_\alpha(\vec{q})$

H is the total energy. $H = T + U$

— holonomic — N = # of degrees of freedom

— $F_\alpha = -\frac{\partial U}{\partial r_\alpha}$ — non-constrained force.

$\mathcal{L} = \mathcal{L}(\dot{q}, q, t) = T - U(q,t)$ $H = \sum_i p_i \dot{q}_i - \mathcal{L}$

$p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$ → for holonomic system can solve for $\dot{q}_i(\vec{q}, \vec{p}, t)$

$$H = \sum_i p_i \dot{q}(\vec{q}, \vec{p}, t) - \mathcal{L}(q, \dot{q}(p,q,t), t)$$

$$\frac{\partial H}{\partial q_i} = \sum_i p_i \frac{\partial \dot{q}_i}{\partial q} - \frac{\partial \mathcal{L}}{\partial q_i} - \sum \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \frac{\partial \dot{q}_i}{\partial q_i} = -\frac{d}{dt} p_i = -\dot{p}_i$$

$$\frac{\partial H}{\partial p_i} = \dot{q}_i + \sum_i p_i \frac{\partial \dot{q}_i}{\partial p_i} - \sum \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \frac{\partial \dot{q}_i}{\partial p_i} = \dot{q}_i$$

$$\frac{dH}{dt} = \sum_i \left[\frac{\partial H}{\partial q_i} \dot{q}_i + \frac{\partial H}{\partial p} \dot{p}_i \right] + \frac{\partial H}{\partial t} \rightarrow \frac{dH}{dt} = \frac{\partial H}{\partial t}$$

$$-\dot{p}_i \dot{q}_i + \dot{q}_i \dot{p}_i = 0$$

- if $\frac{\partial H}{\partial t} = 0$ H is conserved.

- Central Force: angular momentum is conserved. Simplify to 2D problem

$T = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\varphi}^2)$ $H = T + U(r)$

conjugate momentum: $p_r = \frac{\partial T}{\partial \dot{r}} = m\dot{r}$ $p_\varphi = \frac{\partial T}{\partial \dot{\varphi}} = mr^2\dot{\varphi}$

$$H = T + U = \frac{p_r^2}{2m} + \frac{p_\varphi^2}{2mr^2} + U(r)$$

- $\frac{\partial H}{\partial p}$: $\frac{\partial H}{\partial p_r} = \frac{p_r}{m} = \dot{r}$ $\frac{\partial H}{\partial p_\varphi} = \frac{p_\varphi}{mr^2} = \dot{\varphi}$

- $\frac{\partial H}{\partial q}$; $-\frac{\partial H}{\partial r} = \dot{p}_r = \frac{p_\varphi^2}{mr^3} - \frac{\partial U}{\partial r}$ $-\frac{\partial H}{\partial \varphi} = \dot{p}_\varphi = 0$