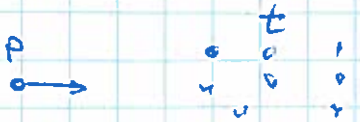
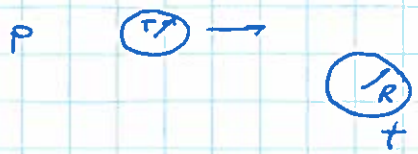
L24 Mean free path.



p – projectile
t – target

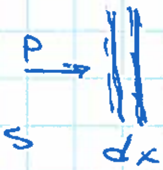
mean free path – average distance p travels before hit t

Example: gas of billiard balls.



$\sigma = \pi (r+R)^2$

dx – thin slice of gas



$n_t = \frac{N}{V} dx \qquad d\mathbb{P} = n_t \sigma = \frac{N\sigma}{V} dx = -[\mathbb{P}(x+dx) - \mathbb{P}(x)]$

$\mathbb{P}(x) - \mathbb{P}(x+dx) = \mathbb{P}(x) \cdot \frac{N\sigma}{V} dx$ ← product of 2 probabil.

probability to travel distance x; probability to trave dist. dx.

$\rightarrow \; -d\mathbb{P} = \mathbb{P} \frac{N\sigma}{V} dx \;\rightarrow\; -\frac{d\mathbb{P}}{dx} = \mathbb{P}(x) \frac{N\sigma}{V}$

or $\int \frac{dP}{\mathbb{P}} = -\int \frac{N\sigma}{V} dx = \ln P(x) = -\frac{N\sigma}{V} x \qquad P(x) = e^{-\frac{N\sigma}{V} x}$

M.F.P. $\lambda = -\int_0^\infty x \frac{dP}{dx} dx = \int x \frac{N\sigma}{V} e^{-\frac{N\sigma}{V} x} dx = \frac{V}{N\sigma}$

So $P(x) = e^{-x/\lambda}$

if a beam of N_0 projectiles is hitting target.

$N(x) = N_0 e^{-x/\lambda}$ are not scattered at dist x.

Air molecules free path: $V = 22.4$ litres

$\sigma = 4\pi \cdot (1.5 \cdot 10^{-10} m)^2 \qquad N = 6 \cdot 10^{23}$ (Avogadro)

$\lambda = 130\,nm \gg 3\,nm \gg 0.3\,nm$

Generalization of a cross-section σ

projectile may interact with target in different ways:

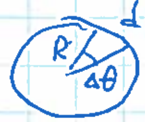
σ_s – scattering

σ_i – ionization $\qquad \sigma_{tot} = \sigma_s + \sigma_i + \sigma_c$

σ_c – capture

...

L24 Differential cross-section.

Solid angle  $dl = R\,d\theta$  $dA = R^2 d\Omega$

full solid angle $= 4\pi$

in spherical coordinates 

$$dA = r\,d\theta\; r\sin\theta\, d\varphi$$

$$d\Omega = \sin\theta\, d\theta\, d\varphi$$

in direction θ, φ. count scattering in solid angle $d\Omega$

$$dN_{sc}(d\Omega) = N_{inc}\, n_{tar}\, d\sigma(d\Omega)$$

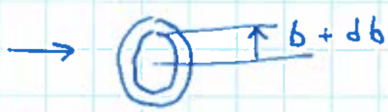
$d\sigma$ - is fraction of cross-section in $d\Omega$

$$d\sigma = \frac{d\sigma}{d\Omega} d\Omega \qquad dN_{sc} = N_{in}\, n_{tar} \frac{d\sigma}{d\Omega}\, d\Omega$$

or.
$$\frac{dN_{sc}(\theta\varphi)}{d\Omega} = N_{in}\, n_{tat} \frac{d\sigma(\theta,\varphi)}{d\Omega}$$

$$\sigma_{total} = \int_0^{4\pi} \frac{d\sigma}{d\Omega}\, d\Omega = \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\varphi\; \frac{d\sigma(\theta\varphi)}{d\Omega}$$

* Calculation. of $\frac{d\sigma}{d\Omega}$



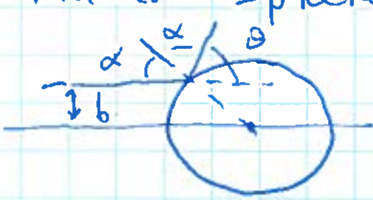
$$d\sigma = 2\pi\, b\, db$$

$$d\Omega = 2\pi \sin\theta\, d\theta \quad \text{(axial target)}$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

need to calculate projectiles trajectory and scattering angle.

Hard sphere:



$$b = R\sin\alpha = R\cos\theta/2$$

$$\frac{d\sigma}{d\sigma} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \frac{R\cos\theta/2}{\sin\theta}\,\frac{R\sin\theta/2}{2} = \frac{R^2}{4}$$

scattering is isotropic $\quad \sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \int \frac{R^2}{4} d\Omega = \frac{R^2}{4} \cdot 4\pi$

$$\sigma = \pi R^2$$