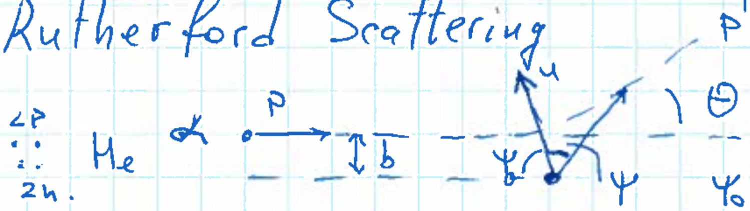
L25 Rutherford Scattering



$\psi_0 = \psi(\infty)$

$F = \frac{kqQ}{r^2} = \frac{\gamma}{r^2}$ Coulomb force. $\theta = \pi - 2\psi_0$

momentum transfer: $\Delta \vec{P} = \vec{P}' - \vec{P}$

Conservation of energy: $p' = p \rightarrow |\Delta P| = 2p \sin\frac{\theta}{2}$

$|\Delta P| = \int F\, dt \qquad \widehat{\Delta P} = \hat{u} \qquad |\Delta P| = \int F_u\, dt$

$F_u = \gamma/r^2 \cos\psi \qquad |\Delta P| = \int \frac{\gamma}{r^2} \cos\psi \frac{d\psi}{\dot\psi} = \int \frac{m\gamma}{m r^2 \dot\psi} \cos\psi\, d\psi$

$m r^2 \dot\psi$ — angular momentum, which is conserved

$m r^2 \dot\psi = pb$

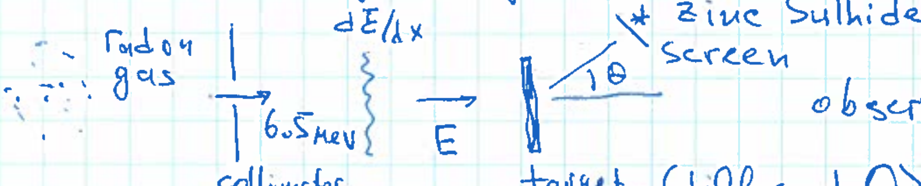
$$|\Delta P| = \int_{-\psi_0}^{\psi_0} \frac{m\gamma}{bp} \cos\psi\, d\psi = 2\frac{\gamma m}{bp} \sin\psi_0 = 2\frac{\gamma m}{bp} \cos\theta/2$$

also $|\Delta P| = 2p \sin\theta/2 \Rightarrow b = \frac{\gamma m}{p^2} \frac{\cos\theta/2}{\sin\theta/2}$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \frac{1}{2\sin\theta/2 \cos\theta/2} \frac{\gamma m}{p^2} \frac{\cos\theta/2}{\sin\theta/2} \cdot \frac{\gamma m}{p^2} \frac{1}{2\sin^2\theta/2} =$$

$$= \left(\frac{2\gamma m}{4p^2} \frac{1}{\sin^2\theta/2}\right)^2 \xrightarrow{\frac{p^2}{2m} = E} \left(\frac{kqQ}{4E \sin\theta/2}\right)^2$$

Experiments by Geiger & Marsden.



(different Q)

observe "scintillations"

fully in agreement with nuclear model of an atom.

CM & Lab Frames.

$\vec{r} = \vec{r}_1 - \vec{r}_2$

Lab: $p \rightarrow$ - - - $\circ$ $p = 0$ (m_1, m_2)

CM: $\xrightarrow{p_1}$ m_1 $\xleftarrow{p_2}$ m_2 $\vec{p}_1 = -\vec{p}_2$

$$\vec{r}_1 = \vec{R} + \frac{m_2}{M}\vec{r} \qquad \vec{r}_2 = \vec{R} - \frac{m_1}{M}\vec{r} \qquad M = m_1 + m_2$$

$$\mathcal{L} = \frac{1}{2}M\dot{R}^2 + \frac{1}{2}\mu\dot{r}^2 - U(r) \qquad \mu = \frac{m_1 m_2}{M}$$

$$P_1 = -P_2 \rightarrow m_1\dot{r}_1 = \frac{m_1 m_2}{M}\dot{r} = \mu\dot{r}$$

$$P_1 = \mu\dot{r} \qquad m_2\dot{r}_2 = -\frac{m_1 m_2}{M}\dot{r} = -\mu\dot{r}$$

total cross-section $\sigma_{lab} = \sigma_{cm}$.

diff cross-section: $\left(\frac{d\sigma}{\partial\Omega}\right)_{cm} d\Omega_{cm} = \left(\frac{\partial\sigma}{\partial\Omega}\right)_{Lab} d\Omega_{lab}$.

$$\left(\frac{d\sigma}{d\Omega}\right)_{Lab} = \left(\frac{d\sigma}{d\Omega}\right)_{CM}\frac{d\Omega_{cm}}{d\Omega_{lab}} = \left(\frac{d\sigma}{d\Omega}\right)_{CM}\frac{d(\cos\theta_{cm})}{d(\cos\theta_{Lab})}$$