L30 3D Rotation. 3D space.

$S \to S'$
$\{\hat{e}_i\}$ $\{\hat{e}'_i\}$

$S: \ r = x_1\hat{e}_1 + x_2\hat{e}_2 + x_3\hat{e}_3$
$S': \ r' = x'_1\hat{e}'_1 + x'_2\hat{e}'_2 + x'_3\hat{e}'_3$

$$x' = \vec{e}'\vec{x} = \sum_i (\hat{e}'_i \hat{e}_i) X = R X$$

Rotation: norm $|x'| = |X|$ conserved

$x_1^2 + x_2^2 + x_3^2 = (x \cdot x) = (\vec{x}' \cdot \vec{x}')$ – invariant scalar product.

Lorentz transformation

4 vector: $S(\vec{r}, ct) = (x_1, x_2, x_3, x_4)$ $\quad x_4 = ct$

$S'(\vec{r}', ct') = (x'_1, x'_2, x'_3, x'_4)$

$$x'_1 = \gamma(x_1 - vt) = \gamma x_1 - \gamma\beta x_4$$
$$x'_2 = x_2 \qquad\qquad x' = \Lambda X$$
$$x'_3 = x_3$$
$$x'_4 = \gamma\left(t - \frac{xv}{c^2}\right)c = -\gamma x_1 \beta + \gamma x_4$$

$$\Lambda = \begin{vmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{vmatrix}$$ standard boost.

$$\Lambda_R = \left|\begin{array}{c|c} R & \begin{matrix}0\\0\\0\end{matrix} \\ \hline 0\ 0\ 0 & 1 \end{array}\right|$$ – rotation in 3D space.

The invariant scalar product:

$S = (x, x) = x_1^2 + x_2^2 + x_3^2 - x_4^2 = r^2 - c^2t^2$ – length square or norm squared

$(x, y) = (x', y')$ – scalar product of (x, y) is conser[ved]

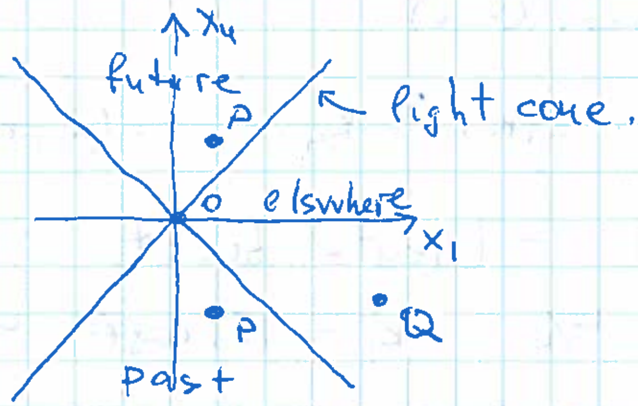
$S' = S$ difference wrt 3D rotation:

S can be negative!

for light: $S = r^2 - c^2t^2 = 0 = r'^2 - c^2t'^2$

speed of light is constant in any ref. frame.

L 30 Light cone.



a) future & past $r^2 < c^2t^2$

$t_P > t_o$

$x \cdot x < 0 \Rightarrow (x', x')$

P is absolute future of O (or past)

b) elswhere : $r^2 > c^2t^2$

$(x \cdot x) > 0$

$t_Q > t_o \quad t_Q = t_o \quad t_Q < t_o$

$x \cdot x < 0$ – time-like vector x

$x \cdot x > 0$ – space-like vetor x

rest frame : $x = (0, 0, 0, ct)$

if a particle moves with V $\quad dx^2 = (v^2 - c^2)dt^2$

or $\quad dx^2 = c^2 dt^2/\gamma^2$

The Quotient Rule : not every combination of 4 numbers is a 4-vector

if x is a 4-vector and $k = (k_1, k_2, k_3, k_4)$ and if $x \cdot k = x_1 k_1 + x_2 k_2 + x_3 k_3 - x_4 k_4$ is invariant (same value in all frames) than k is a 4-vector.

$k \cdot x = k' \cdot x' = (\Lambda k)(\Lambda x) = (\Lambda k)\, x'$

$k' = \Lambda k$ transforms as 4-vector.

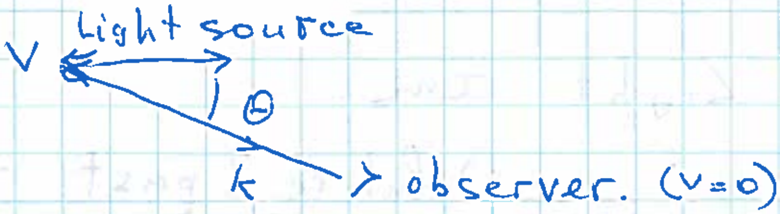
* Doppler effect. plane wave $\varphi = A \cos(\vec{k} \cdot \vec{x} - \omega t - \delta)$

$\vec{K}$ – wave vector $|k| = 4\pi/\lambda$ λ – wavelength.

$\omega = 2\pi\nu \quad \nu$ – frequency

V = speed of wave $= \omega/|k| = \lambda\nu = c$ for light.

$\vec{K} \cdot \vec{x} - \omega t = k \cdot x \quad k = (\vec{K}, \omega/c)$ – 4-vector.



for light $\qquad \omega/c = |K| = K_4 \qquad k = (\vec{k}, k_4)$

$$k_4' = \gamma(k_4 - \beta k_1) \quad \text{— lorentz boost.}$$

$$k_4' = \omega'/c \qquad k_4 = \omega/c \qquad k_1 = |\vec{k}|\cos\theta = \omega/c \cos\theta$$

$$\omega' = \gamma\omega(1 - \beta\cos\theta)$$

$$\omega = \frac{\omega_0}{\gamma(1-\beta\cos\theta)}$$

ω_0 – light frequency at the rest frame of the source.