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Relativistic Dynamics

* mass: The mass m of an object is defined in Rest Frame (RF) — 4-scalar
* world line is defined by 4-vector $x = (\vec{x}(t), ct)$
* proper time is measured in RF

dt_0: $dx_0 = x_0(t+dt_0) - x_0(t) = (0, 0, 0, c\,dt_0)$

moving frame: $dx = (v\,dt, c\,dt)$

Since $dx = dx_0 \rightarrow -c^2 dt_0^2 = (v^2 - c^2)\,dt \rightarrow dt_0 = dt/\gamma$

dt_0 is the proper time interval — 4-scalar.

* 4-velocity $u = \frac{dx}{dt_0} = \left(\frac{dx}{dt_0}, c\frac{dt}{dt_0}\right)$ — 4-vector.

replace $dt_0 \rightarrow dt/\gamma \rightarrow u = \gamma\left(\frac{dx}{dt}, c\right)$ — 4-vector.

the spatial part of u is not velocity v
it becomes so when $v \ll c$

* Relativistic momentum
  - should produce $p = mv$ in classical limit
  - should be conserved for isolated system in all frame

if classical momentum $\sum p_i = \sum m_i v_i$ is conserved in S,
it may not be conserved in S'

- relativistic momentum: $p = mu = (\gamma m v, \gamma m c)$

m — 4-scalar u — 4-vector $\rightarrow$ QR $\rightarrow$ P is 4-vector.

conservation law: $\sum P_{in} = \sum P_{out}$
includes both conservation of 3D momentum
and energy. $E = \gamma m c^2 = P_4 c$

$$P = (\vec{P}, E/c)$$ — momentum energy 4-vector.

classical limit: $E \approx mc^2 + \frac{1}{2} m v^2$

classical conservation of enery:

$\sum_i E^i_{in} = \sum_i E^i_{fin} \rightarrow \sum_{in}(m_i c^2 + \frac{1}{2} m_i v_i^2) = C_{in} + T_{in} = C_{fin} + T_{fin}$

conservation of mass $C_{in} = C_{fin} \rightarrow$ kinetic energy T is conserved.

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inelastic collision: $T_{in} \neq T_{fin}$.

→ $C_{in} \neq C_{fin}$ mass is not conserved if we believe in conservation of energy $E_{in} = E_{fin}$.

$$\Delta m c^2 = -\Delta T \to \Delta M = -\Delta T / c^2$$ – usually small!

Rest energy: $E = \gamma m c^2$ $\gamma \to 1$ $\boxed{E = mc^2}$

anihilation of $e^+ e^- \to 2\gamma$ (radiation)

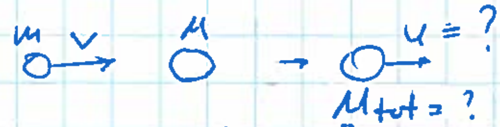
mass → radiation.

$E = mc^2 + T$ – definition of kinetic energy

$\gamma m c^2 = mc^2 + T$ $T = mc^2(\gamma - 1)$

$\beta E = pc$; $p \cdot p = -mc^2$; $E^2 = (mc^2)^2 + (pc)^2$

Collisions: Example 15.9



$P_{in}^2 = (P_m + P_M)^2 = P_m^2 + P_M^2 + 2 P_m P_M = -mc^2 - Mc^2 + 2 P_m P_M$

$P_m = (p, \gamma m c)$ · $(P_M = 0, Mc)$ → $2 P_m P_M = (0 - \frac{E_m}{c} \cdot Mc) 2$

$P_{in}^2 = -mc^2 - Mc^2 - 2 E_m M$

$P_{fin} = (0, M_{tot})$ $P_{fin}^2 = -M_{tot}^2 c^2$

$M_{tot} = \sqrt{m^2 + M^2 + 2 \frac{E_m}{c^2} M}$ classical limit $M_{tot} = (m + M)$

$E_m = mc^2$

$$v = \frac{\vec{p}_{fin} c^2}{E_{fin}} = \frac{\vec{p} c^2}{E_m + Mc^2} =$$

CM frame: $\vec{P} = \Sigma \vec{p} = 0$ $P \cdot P = (0, 0, 0, P_4)$