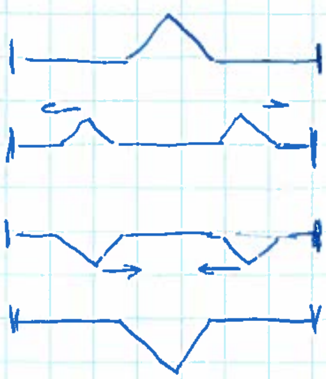
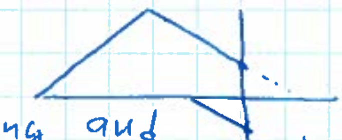
L34 Triangular wave on finite string.



wave hits the boundary.



incoming and reflected waves should cancel each other on the boundary

* 3D wave equation

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0 \rightarrow \frac{\partial^2 u}{\partial t^2} - c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = 0$$

$c = \sqrt{\frac{BM}{\rho}}$ BM – bulk modulus (resistanse to compressi[on]

ρ – density.

Laplacian: $\nabla^2 = \vec{\nabla} \cdot \vec{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

$\frac{\partial^2 u}{\partial t^2} - c^2 \nabla^2 u$ $u(x, y, z)$ – disturbance in 3D system

use p instead of u

Plane waves: – simplest solution of 3D wave eq.

$P(\vec{r}, t) = P(x, t) \rightarrow \frac{\partial^2 P}{\partial t^2} - c^2 \frac{\partial^2 P}{\partial x^2} = 0$

solution: $p = f(x - ct) + g(x + ct)$

or in general: $P = f(\vec{n} \cdot \vec{r} - ct)$ – $\vec{n}$ – direction of travel.

important case: $P = \cos(k \cdot (\vec{n} \cdot \vec{r} - ct))$

$k = \frac{2\pi}{\lambda}$ – wave number. $k\vec{n} = \vec{k}$

$$P = \cos(\vec{k} \cdot \vec{r} - \omega t)$$

L34

Boundary Conditions.

strings are attached to something →
boundary conditions at $x=0$ and $X=L$
L – length of the string.

So $0<x<L$ $\quad u(0,t) = u(L,t) = 0$

find solutions of $\frac{\partial^2 y}{\partial t^2} - c^2 \frac{\partial^2 y}{\partial x^2} = 0$

start with harmonic wave $u(x,t) = A(x)\cos(\omega t - \delta)$

substitute into wave equation

$$-\omega^2 A(x)\cos(\omega t-\delta) - c^2 \frac{\partial^2 A}{\partial x^2}\cos(\omega t - \delta) = 0$$

$$\frac{\partial^2 A}{\partial x^2} + k^2 A(x) = 0 \qquad k^2 = \omega/c$$

ordinary diff. eq. $\quad A(x) = a\cos(kx) + b\sin(kx)$

$A(0) = A(L) = 0 \rightarrow u(x,t) = A_0 \sin(kx)\cos(\omega t - \delta)$

$u(0,t) = u(L,t) = 0 \rightarrow \quad k = k_n = n\frac{\pi}{L} \quad (n = 1,2,3 \ldots)$

$$\omega = \omega_n = n\frac{\pi c}{L}$$

similar to system of coupled oscillators
general solution is a superposition of
normal modes with frequencyes ω_n

ω_1 – fundamental mode
$\omega_{i>1}$ – obertones

General Solution:

$$u_n(x,t) = \sin k_n x\,(B_n\cos\omega_n t + C_n \sin\omega_n t)$$

$$u(x,t) = \sum_n u_n(x,t)$$

L34 Spherical waves:

$p = p(r,t)$ — spherically symmetric

$$\nabla^2 \psi = \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2}$$

spherical symmetry $\nabla^2\psi = \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi)$

$$\frac{\partial^2 p}{\partial t^2} - c^2 \frac{1}{r}\frac{\partial^2(rp)}{\partial r^2} = 0 = \frac{\partial^2(pr)}{\partial t^2} - c^2 \frac{\partial^2(pr)}{\partial r^2} = 0$$

$$r\,p(r,t) = f(r-ct) + g(r+ct)$$

$$p(r,t) = \frac{1}{r}\left[f(r-ct) + g(r+ct)\right]$$

amplitude of the wave decrease as $1/r$

intensity $\sim 1/r^2$

Volume & surface forces.

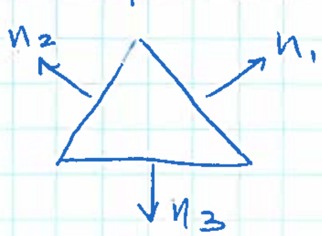
Forces on an element of volume : dV

Volume forces: $F \sim dV$ like gravity

$\vec{F_g} = \rho\, dV \vec{g}$

surface forces: pressure, tension, shear

Isotropic pressure in fluids. (Shear force $= 0$)



$F_i = -P_i n_i dA_i$

$\sum \vec{F_i} + \vec{F}_{vol} = m\vec{a} \rightarrow \sum \vec{F_i} = m\vec{a} - \vec{F}_{vol}$

if we shrink the prizm by λ

$\lambda^2 \sum \vec{F_i} = \lambda^3 (m\vec{a} - \vec{F}_{vol})$

$\sum \vec{F_i}$ should be $\emptyset$ since $|F_1| = |F_2|$ then $P_1 = P_2$ regardless on direction n_1, n_2