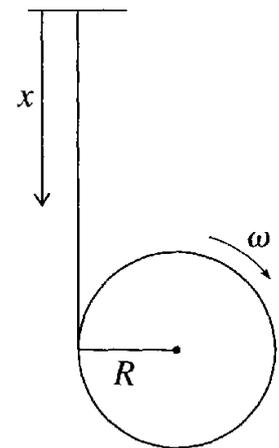


PH4222, Section 3801, Spring 2014, Homework 3

Due at the start of class on Friday, January 31. Half credit will be available for late homework submitted no later than the start of class on Monday, February 3.

Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working.

1. Consider the crude model of a yoyo shown in the figure. A massless string is suspended vertically from a fixed point and the other end is wrapped several times around a uniform cylinder of mass m and radius R . When the cylinder is released it moves vertically down, rotating as the string unwinds.



- Write down the Lagrangian, using the distance x as your generalized coordinate.
- Find the Lagrange equation of motion and show that the cylinder accelerates downward with $\ddot{x} = 2g/3$.

Hints: Remember that the total kinetic energy of a body like the yoyo is $T = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, where v is the velocity of the center of mass (CM), I is the moment of inertia (for a uniform cylinder, $I = \frac{1}{2}mR^2$) and ω is the angular velocity about the CM. Express ω in terms of \dot{x} .

2. A mass m_1 rests on a frictionless horizontal table and is attached to a massless string. The string runs horizontally to the edge of the table, where it passes over a massless, frictionless pulley and then hangs vertically down. A second mass m_2 is now attached to the bottom end of the string. For your generalized coordinates, use the displacement of each mass from rest.

- Write down the Lagrangian for the system, including the constraint.
- Find the Euler-Lagrange equations of motion for the system.
- Identify the Lagrange multiplier in terms of the tension acting on each mass.
- Solve the E-L equations for the accelerations of the blocks.

3. A bar of soap (mass m) is at rest on a frictionless rectangular plate that rests on a horizontal table. At time $t = 0$, I start raising one edge of the plate so that the plate pivots about the opposite edge with constant angular velocity ω , and the soap starts to slide toward the downhill edge.

- a) Show that the equation of motion for the soap has the form $\ddot{x} - \omega^2 x = -g \sin \omega t$, where x is the soap's distance from the downhill edge.
- b) Solve this for $x(t)$, given that $x(0) = x_0$. *Hint:* You will need to find both the general homogeneous solution and a particular solution, for which you can try $x = A \sin \omega t$ and solve for A.

4. Two masses m_1 and m_2 move in a plane and interact by a potential energy $U(r) = \frac{1}{2}kr^2$. Write down their Lagrangian in terms of the CM and relative positions \mathbf{R} and \mathbf{r} , and find the equations of motion for the coordinates X, Y and x, y . Describe the motion and find the frequency of the relative motion.

- a) Write down their Lagrangian in terms of the CM and relative positions \mathbf{R} and \mathbf{r} .
- b) Find the equations of motion for the coordinates X, Y and x, y .
- c) Describe the motion and find the frequency of the relative motion.

5. A particle of mass m is constrained to move on the inside surface of a smooth cone of half-angle α . The particle is subject to a gravitational force.

- a) Choose a set of generalized coordinates and indicate the constraints.
- b) Write down the Euler-Lagrange equations of motion.
- c) Show that the particle can have stable circular motion.
- d) Find the frequency for its periodic motion.

