PHY 4324 December 2, 2013

Exam #2

. . . such names as Laplace, Maxwell and Einstein — do **they** *mean anything to you? . . . Physicists, according to our not-so-reliable historians. Responsible for the rapid rise of the European-American culture . . .*

A Canticle for Leibowitz

(1) Suppose you take a plastic ring of radius a and glue charge on it, so that the linear charge density is $\lambda_0|\sin(\phi/2)|$. Then you spin the loop about its axis at an angular velocity of ω.

a) What are the (time dependent) charge density $\rho(s, \phi, z, t)$ and current density $\vec{J}(s, \phi, z, t)$? **(10 points)**

b) What is the retarded time $t_{\text{ret}}(\vec{r}, t; \vec{r}')$ from any point $\vec{r}' = a\hat{s}$ on the ring to the center of the ring $\vec{r} = 0$? **(10 points)**

c) What is the retarded scalar potential at the center of the ring? **(10 points)**

d) What is the retarded vector potential at the center of the ring? **(10 points)**

(2) A particle of mass m and charge q is attached to a spring with force constant k, hanging from the ceiling (see figure). Its equilibrium position is a distance h above the floor. It is pulled down a distance d below equilibrium and released, at time $t = 0$.

a) Under the usual assumptions ($d \ll \lambda \ll h$), calculate the intensity of the radiation hitting the floor, as a function of the distance R from the point directly below q. Neglect radiative damping of the oscillator. *Note:* The intensity here is the average power per unit area of *floor*. **(20 points)**

b) Assume the floor extends to infinity. What is the average energy per unit time striking the entire floor? If you cannot get a), just explain how to do it and guess the result using dimensional analysis. **(20 points)**

c) Because it is losing energy in the form of radiation, the amplitude of the oscillation will gradually decrease. Assume the fraction of the total energy lost in one cycle is very small. After what time τ has the amplitude been reduced to d/e? If you cannot get a) and/or b), just explain how to do it and guess the result using dimensional analysis. **(10 points)**

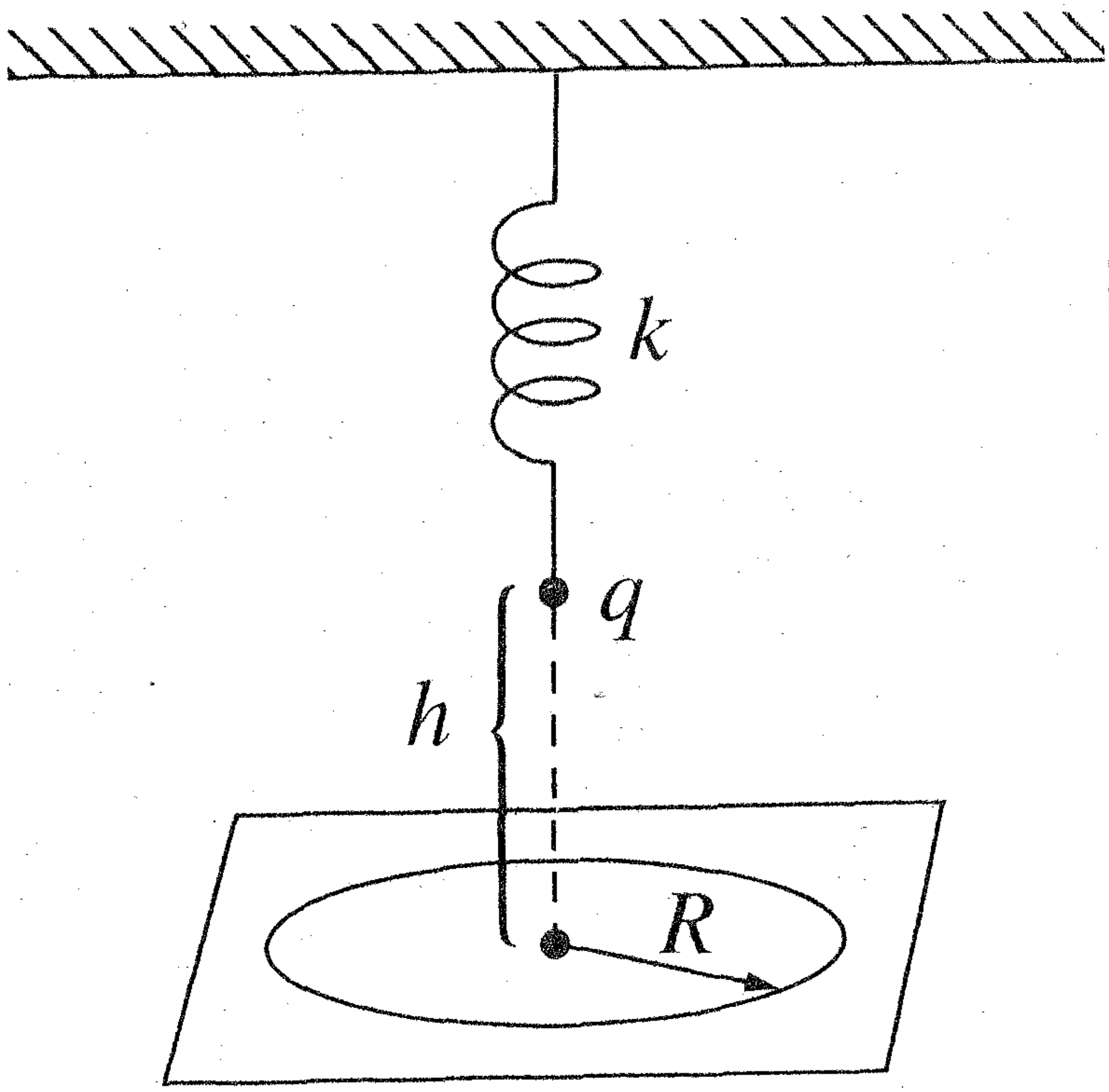
(3) A charge q is released from rest at the origin, in the presence of a uniform electric field $\vec{E} = E_0\hat{z}$ and a uniform magnetic field $\vec{B} = B_0\hat{x}$. Assume $E_0 < cB_0$.

a) Find the trajectory $\vec{r}(t)$ of the particle in the nonrelativistic limit. **(25 points)**

b) Find the velocity $\vec{v}$ of an observer S' whose electric field $\vec{E}'$ is zero. Also find the magnetic field $\vec{B}'$ in this frame. **(25 points)**

c) Use the Lorentz force law in the S' frame to find the trajectory $\vec{r}'(t')$ of the charge, including relativistic effects. If you cannot get part b), just solve the problem assuming $\vec{B} = B_0'\hat{x}$ and an initial velocity in the $\hat{y}$ direction. **(25 points)**

d) Give the charge's S frame trajectory $\vec{r}\Big(t(t')\Big)$ as a function of the S' frame time, and write down the transcendental equation which determines t' as a function of t. **(25 points)**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