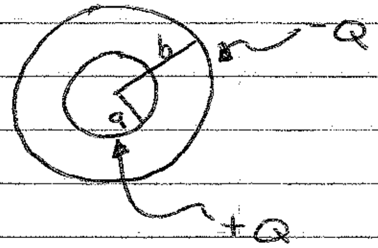
8/21/13
R.P.Woodard

Solution to Problem 7.1



(a) * solve the electrostatics problem just as if no current flows

* for $a<r<b \rightarrow \vec{E} = \frac{Q\hat{r}}{4\pi\epsilon_0 r^2} \rightarrow V(r) = \frac{Q}{4\pi\epsilon_0 r}$

* the potential difference from $r=a$ to $r=b$ is

$$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right] \rightarrow \frac{Q}{4\pi\epsilon_0} = \frac{V}{\frac{1}{a}-\frac{1}{b}} = \frac{aV}{1-a/b} \rightarrow \vec{E} = \frac{aV}{1-a/b}\frac{\hat{r}}{r^2}$$

* now use Ohm's Law $\rightarrow \vec{J} = \sigma\vec{E} = \frac{\sigma a V}{1-a/b}\frac{\hat{r}}{r^2}$

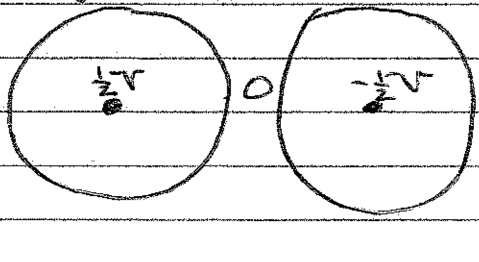
* the total current is the integral of $\vec{J}\cdot d\vec{a}$ over a surface of constant r

$$\rightarrow \boxed{I = \oint d\vec{a}\cdot\vec{J} = \frac{4\pi\sigma a V}{1-a/b}}$$

(b) * $V = IR \rightarrow \boxed{R = \frac{1-a/b}{4\pi\sigma a V}}$

(c) * for large b the potential is almost zero & making b larger doesn't increase the potential difference by much

* now imagine the two spheres as each surrounded by a large circle at potential 0



* to keep the potential difference between them we must make one at potential $+\frac{1}{2}V$ & the other at $-\frac{1}{2}V$

∴ the current which flows is $\boxed{I = 2\pi\sigma a V}$

& of course the resistance is $\boxed{R = \frac{1}{2\pi\sigma a}}$