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Solution to Problem 9.3

* just use $\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$ backwards & forth

* $A_1\cos(kz-\omega t+\delta_1)+A_2\cos(kz-\omega t+\delta_2)$

$$=\cos(kz-\omega t)\left[A_1\cos(\delta_1)+A_2\cos(\delta_2)\right]-\sin(kz-\omega t)\left[A_1\sin(\delta_1)+A_2\sin(\delta_2)\right]$$

* $\left[A_1\cos(\delta_1)+A_2\cos(\delta_2)\right]^2+\left[A_1\sin(\delta_1)+A_2\sin(\delta_2)\right]^2$

$$=A_1^2+2A_1A_2\cos(\delta_1-\delta_2)+A_2^2\equiv A_3^2$$

$\therefore$ $A_1\cos(kz-\omega t+\delta_1)+A_2\cos(kz-\omega t+\delta_2)=A_3\cos(\overset{kz}{\cancel{\omega t}}-\omega t+\delta_3)$

where
$$A_3\equiv\sqrt{A_1^2+2A_1A_2\cos(\delta_1-\delta_2)+A_2^2}$$
$$\&\ \delta_3\equiv\tan^{-1}\left[\frac{A_1\sin(\delta_1)+A_2\sin(\delta_2)}{A_1\cos(\delta_1)+A_2\cos(\delta_2)}\right]$$

* NB of course you get the same result using complex analysis:

$$A_1\cos(kz-\omega t+\delta_1)+A_2\cos(kz-\omega t+\delta_2)=\mathrm{Re}\left[A_1e^{i(kz-\omega t+\delta_1)}+A_2e^{i(kz-\omega t+\delta_2)}\right]$$

$$=\mathrm{Re}\left\{e^{i(kz-\omega t)}\left[A_1e^{i\delta_1}+A_2e^{i\delta_2}\right]\right\}$$

* $\left|A_1e^{i\delta_1}+A_2e^{i\delta_2}\right|^2=A_1^2+2A_1A_2\cos(\delta_1-\delta_2)+A_2^2\equiv A_3^2$