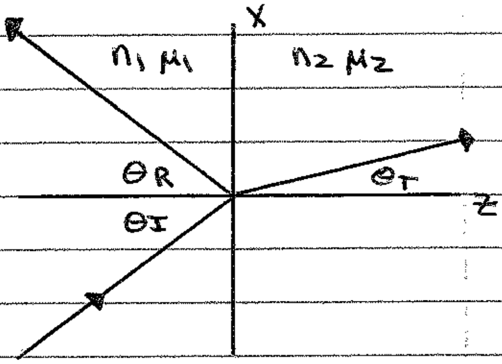
9/12/13
R.P.Woodard

For $\mu_1=\mu_2$
$$\varsigma \equiv \frac{n_2}{n_1}\frac{\cos\theta_T}{\cos\theta_I} = \sqrt{1+\left(\frac{n_2^2-n_1^2}{n_1^2}\right)\sec^2\theta_I}$$
$$\begin{cases} \text{Case ① } n_2>n_1 \Rightarrow \varsigma>1\ \forall\theta \Rightarrow E_R<0\ \forall\theta \\ \text{Case ② } n_2<n_1 \Rightarrow \varsigma<1\ \forall\theta \Rightarrow E_R>0\ \forall\theta \end{cases}$$

Solution to Problem 9.16



$\boxed{\text{Kinematics}}$

$*\ \theta_R=\theta_I$

$*\ n_1\sin\theta_I = n_2\sin\theta_T$

$\therefore\ \vec{k}_I = \frac{n_1\omega}{c}(\sin\theta_I\,\hat{x}+\cos\theta_I\,\hat{z})$

$\vec{k}_R = \frac{n_1\omega}{c}(\sin\theta_I\,\hat{x}-\cos\theta_I\,\hat{z})$

$\vec{k}_T = \frac{n_2\omega}{c}(\sin\theta_T\,\hat{x}+\cos\theta_T\,\hat{z})$

$\boxed{\text{Polarization} \perp \text{to plane of incidence}}$

$*\ \vec{E}_{I0}=E_I\,\hat{y} \Rightarrow \vec{B}_{I0}=\frac{k_I}{\omega}\hat{k}_I\times\vec{E}_{I0} = \frac{n_1E_I}{c}(\sin\theta_I\,\hat{z}-\cos\theta_I\,\hat{x})$

$*\ \vec{E}_{R0}=E_R\,\hat{y} \Rightarrow \vec{B}_{R0}=\frac{k_I}{\omega}\hat{k}_R\times\vec{E}_{R0} = \frac{n_1E_R}{c}(\sin\theta_R\,\hat{z}+\cos\theta_R\,\hat{x})$

$*\ \vec{E}_{T0}=E_T\,\hat{y} \Rightarrow \vec{B}_{T0}=\frac{k_T}{\omega}\hat{k}_T\times\vec{E}_{T0} = \frac{n_2E_T}{c}(\sin\theta_T\,\hat{z}-\cos\theta_T\,\hat{x})$

$\boxed{\text{BC from Eqn 9.101}}$

(i) $\epsilon_1[\vec{E}_{I0}+\vec{E}_{R0}]_z=\epsilon_2(\vec{E}_{T0})_z \Rightarrow 0=0$

(ii) $[\vec{B}_{I0}+\vec{B}_{R0}]_z=(\vec{B}_{T0})_z \Rightarrow \frac{n_1}{c}\sin\theta_I(E_I+E_R)=\frac{n_2}{c}\sin\theta_T\,E_T \Rightarrow \boxed{E_I+E_R=E_T}$

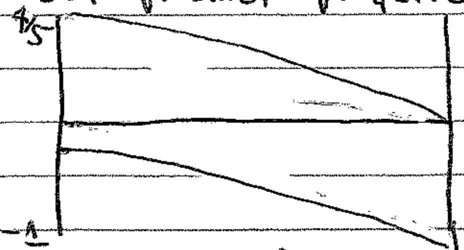
(iii) $[\vec{E}_{I0}+\vec{E}_{R0}]_{x,y}=[\vec{E}_{T0}]_{x,y} \Rightarrow E_I+E_R=E_T$ (same as ii)

(iv) $\frac{1}{\mu_1}[\vec{B}_{I0}+\vec{B}_{R0}]_{x,y}=\frac{1}{\mu_2}[\vec{B}_{T0}]_{x,y} \Rightarrow \frac{n_1\cos\theta_I}{\mu_1 c}(E_I-E_R)=\frac{n_2\cos\theta_T}{\mu_2 c}E_T \Rightarrow \boxed{E_I-E_R=\frac{\mu_1n_2\cos\theta_T}{\mu_2n_1\cos\theta_I}E_T}$

$*$ Define $\varsigma=\frac{\mu_1n_2\cos\theta_T}{\mu_2n_1\cos\theta_I} \Rightarrow \begin{cases}E_I+E_R=E_T\\E_I-E_R=\varsigma E_T\end{cases} \Rightarrow \begin{cases}E_R=\left(\frac{1-\varsigma}{1+\varsigma}\right)E_I\\E_T=\frac{2}{1+\varsigma}E_I\end{cases}$

$\boxed{\text{Set } \frac{n_2}{n_1}=3/2\ \&\ \mu_1=\mu_2} \Rightarrow \varsigma=\frac{3}{2}\frac{\cos\theta_T}{\cos\theta_I}$

$*\ \cos\theta_T=\sqrt{1-\sin^2\theta_T}=\sqrt{1-\frac{4}{9}\sin^2\theta_I}=\frac{1}{3}\sqrt{5+4\cos^2\theta_I} > \cos\theta_I \Rightarrow E_R<0$



$$\frac{E_T}{E_I}=\frac{2}{\sqrt{\frac{5}{4}\sec^2\theta_I+1}+1} \xrightarrow{\theta_I=0} \frac{4}{5}$$

$$\frac{E_R}{E_I}=-\left[\frac{\sqrt{\frac{5}{4}\sec^2\theta_I+1}-1}{\sqrt{\frac{5}{4}\sec^2\theta_I+1}+1}\right] \longrightarrow -\frac{1}{5}$$

$*\ \vec{S}_{I0}=\frac{1}{\mu_1}\vec{E}_{I0}\times\vec{B}_{I0}=\frac{n_1E_I^2}{\mu_1 c}\hat{k}_I,\ \vec{S}_{R0}=\frac{n_1E_R^2}{\mu_1 c}\hat{k}_R\ \&\ \vec{S}_{T0}=\frac{n_2E_T^2}{\mu_2 c}\hat{k}_T$

$\therefore\ R\equiv-\frac{\hat{z}\cdot\vec{S}_{R0}}{\hat{z}\cdot\vec{S}_{I0}}=\left(\frac{1-\varsigma}{1+\varsigma}\right)^2\ \&\ T\equiv\frac{\hat{z}\cdot\vec{S}_{T0}}{\hat{z}\cdot\vec{S}_{I0}}=\frac{4\varsigma}{(1+\varsigma)^2}$

$\Rightarrow R+T=1$