1/25/13
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Solution to Problem 9.19

(a) * From eqn (9.126) $\kappa = \frac{1}{d} = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+(\frac{\sigma}{\epsilon\omega})^2}-1\right]^{\frac{1}{2}}$

* For $\sigma \ll \epsilon\omega \Rightarrow \sqrt{1+(\frac{\sigma}{\epsilon\omega})^2} = 1 + \frac{1}{2}(\frac{\sigma}{\epsilon\omega})^2 - \frac{1}{8}(\frac{\sigma}{\epsilon\omega})^4 + \dots$

$\Rightarrow \kappa = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\frac{1}{2}(\frac{\sigma}{\epsilon\omega})^2 - \frac{1}{8}(\frac{\sigma}{\epsilon\omega})^4 + \dots\right]^{\frac{1}{2}} = \sqrt{\epsilon\mu}\,\frac{\sigma}{2\epsilon}\left[1 - \frac{1}{4}(\frac{\sigma}{\epsilon\omega})^2 + \dots\right]^{1/2}$

$\therefore \lim_{\sigma/\epsilon\omega \to 0} d = \sqrt{\frac{\epsilon}{\mu}}\frac{2}{\sigma}$ is independent of ω

* For pure water I find $\left\{\begin{array}{l}\sigma \cong 5.5\times10^{-6}\,\frac{C^2 s}{kg\,m^3}\\ \epsilon \cong 80.1\,\epsilon_0\\ \mu \cong 0.999992\,\mu_0\end{array}\right\} \Rightarrow d \cong 8.7\times10^{3}\,m$

* NB sea water has $\sigma \sim 10^6$ larger $\Rightarrow d \sim 1\,cm$

(b) * From eqn (9.126) again

$$\left\{\begin{array}{l} k = \frac{2\pi}{\lambda} = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+(\frac{\sigma}{\epsilon\omega})^2}+1\right]^{\frac{1}{2}} \\ \kappa = \frac{1}{d} = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+(\frac{\sigma}{\epsilon\omega})^2}-1\right]^{\frac{1}{2}} \end{array}\right\} \xrightarrow[\sigma \gg \epsilon\omega]{} \left\{\begin{array}{l} \omega\sqrt{\frac{\epsilon\mu}{2}} * \sqrt{\frac{\sigma}{\epsilon\omega}} = \sqrt{\frac{1}{2}\omega\mu\sigma} \\ \omega\sqrt{\frac{\epsilon\mu}{2}} * \sqrt{\frac{\sigma}{\epsilon\omega}} = \sqrt{\frac{1}{2}\omega\mu\sigma} \end{array}\right\}$$

$\therefore \lim_{\frac{\sigma}{\epsilon\omega}\to\infty} d = \frac{\lambda}{2\pi}$

* For $\sigma \cong 10^7\,\frac{C^2 s}{kg\,m^3}$, $\mu \cong \mu_0$ & $\omega = 10^{15}\,Hz \Rightarrow d \cong 10^{-8}\,m = 10\,nm$

* metals are opaque because they reflect light

(c) * to find the relation between $\vec{E}$ & $\vec{B}$ we need to go back to Faraday's law

$$\vec{\nabla}\times\vec{E} + \dot{\vec{B}} = 0 \xrightarrow[\vec{B} = \vec{B}_0 e^{i(\tilde{k}z-\omega t)}]{\vec{E} = \vec{E}_0 e^{i(\tilde{k}z-\omega t)}} \underbrace{}_{i\tilde{k}\hat{z}}\times\vec{E}_0 - i\omega\vec{B}_0 = 0 \Rightarrow \vec{B}_0 = \frac{\tilde{k}}{\omega}\hat{z}\times\vec{E}_0$$

* $\tilde{k} = k + i\kappa \xrightarrow[\sigma \gg \epsilon\omega]{} \sqrt{\omega\mu\sigma}\,e^{i\pi/4} \Rightarrow \vec{B}_0 = \sqrt{\frac{\mu\sigma}{\omega}}\,\hat{z}\times\vec{E}_0\,e^{i\pi/4}$

$\therefore$ the magnetic field lags the electric field by 45°