9/25/13
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Solution to Problem 9.23

* Recall that the model of Ex. 4.1 implies the force on an electron displaced by x is $F = \frac{-e^2 x}{4\pi\epsilon_0 a^3}$
* set this equal to $-m\omega_0^2 x$

$\Rightarrow \boxed{\omega_0 = \frac{e}{\sqrt{4\pi\epsilon_0 m a^3}}}$

* Plugging the #'s gives

$$\omega_0 \cong \frac{1.6\times10^{-19}C}{\sqrt{4\pi(8.85\times10^{-12}\frac{C^2 s^2}{kg\text{-}m^3})(9.1\cdot10^{-31}kg)(5\times10^{-11}m)^3}} \cong \boxed{4.5\times10^{16}\,Hz} \text{ which is } \boxed{UV}$$

* this model gives

$$\vec{P} = \frac{Ne^2}{m} * \frac{1}{\omega_0^2-\omega^2}\vec{E} \Rightarrow n^2 = 1 + \frac{Ne^2}{m\epsilon_0}\frac{1}{\omega_0^2-\omega^2} \cong 1 + \frac{Ne^2}{m\epsilon_0} * \frac{1}{\omega_0^2}\left[1+\frac{\omega^2}{\omega_0^2}\right]$$

* For $n \cong 1 \Rightarrow n \cong 1 + \frac{Ne^2}{2m\epsilon_0\omega_0^2}\left[1+\left(\frac{2\pi c}{\omega_0\lambda}\right)^2\right] \equiv 1 + A\left[1+\frac{B}{\lambda^2}\right]$

$\therefore A = \frac{Ne^2}{2m\epsilon_0\omega_0^2} = 2\pi a^3 N \cong 4.7\times10^{-4}$ cf $A = 1.36\times10^{-4}$

$B = \left(\frac{2\pi c}{\omega_0}\right)^2 \cong 1.8\times10^{-15}\,m^2$ cf $B = 7.7\times10^{-15}\,m^2$

* not bad for such a crude model