10/13/13
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Solutions to Problem 10.16

* Recall that the retarded potentials are

$$V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int d^3r' \, \frac{\rho(\vec{r}\,', t - \frac{1}{c}\|\vec{r}-\vec{r}\,'\|)}{\|\vec{r}-\vec{r}\,'\|} \;\; \& \;\; \vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int d^3r' \, \frac{\vec{J}(\vec{r}\,', t - \frac{1}{c}\|\vec{r}-\vec{r}\,'\|)}{\|\vec{r}-\vec{r}\,'\|}$$

* Recall also that the charge & current densities of a point charge at $\vec{W}(t)$ are

$$\rho(\vec{r},t) = q\,\delta^3(\vec{r}-\vec{W}(t)) \quad \& \quad \vec{J}(\vec{r},t) = q\,\dot{\vec{W}}(t)\,\delta^3(\vec{r}-\vec{W}(t))$$

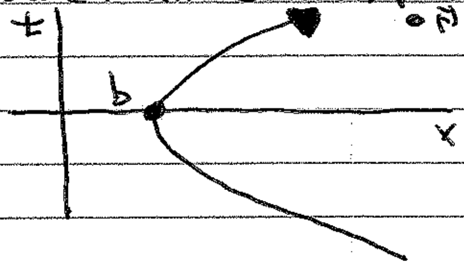
* Recall also that $f(x_0)=0 \rightarrow \delta(f(x)) = \delta(x-x_0)/|f'(x_0)|$

$\therefore$ we must find $\vec{r}\,' \ni \vec{r}\,' = \vec{W}(t - \frac{1}{c}\|\vec{r}-\vec{r}\,'\|)$

* For this problem $\vec{W}(t) = \sqrt{b^2+c^2t^2}\,\hat{x}$ & we assume $\vec{r} = x\hat{x}$ with $x > \sqrt{b^2+c^2t^2}$

$\therefore y' = z' = 0$ & $x' = \sqrt{b^2 + [ct - (x-x')]^2}$

$\rightarrow x' = \frac{1}{2}(x-ct) + \frac{b^2}{2(x-ct)}$



* It's also useful to note

$$\left.\begin{aligned} x - x' &= \tfrac{1}{2}(x+ct) - \frac{b^2}{2(x-ct)} \\ ct_{ret} &= -\tfrac{1}{2}(x-ct) + \frac{b^2}{2(x-ct)} \end{aligned}\right\} \rightarrow \delta\left(x' - \sqrt{b^2+c^2t_{ret}^2}\right) = \frac{\delta\left(x' - \frac{1}{2}(x-ct) - \frac{b^2}{2(x-ct)}\right)}{\left|1 - \frac{ct_{ret}}{\sqrt{b^2+c^2t_{ret}^2}}\right|} = \frac{\delta\left(x' - \frac{1}{2}(x-ct) - \frac{b^2}{2(x-ct)}\right)}{2\left[1+\left(\frac{b}{x-ct}\right)^2\right]^{-1}}$$

* $$V(x\hat{x},t) = \frac{q}{4\pi\epsilon_0}\,\frac{1}{x-x'}\,\frac{1}{\left|1 - \frac{ct_{ret}}{\sqrt{b^2+c^2t_{ret}^2}}\right|} = \frac{q}{4\pi\epsilon_0}\,\frac{\left[1+\left(\frac{b}{x-ct}\right)^2\right]}{x+ct-\left(\frac{b^2}{x-ct}\right)}$$

* $$\vec{A}(x\hat{x},t) = \dot{W}(t_{ret})\,\hat{x} * \frac{V(x\hat{x},t)}{c^2}$$

* $$\dot{W}(t_{ret}) = \frac{c^2 t_{ret}}{\sqrt{b^2+c^2t_{ret}^2}} = c\,\frac{\left[-1+\left(\frac{b}{x-ct}\right)^2\right]}{\left[1+\left(\frac{b}{x-ct}\right)^2\right]}$$

$$\therefore \vec{A}(x\hat{x},t) = \frac{1}{c} * \frac{q}{4\pi\epsilon_0}\,\frac{\left[-1+\left(\frac{b}{x-ct}\right)^2\right]}{x+ct-\left(\frac{b}{x-ct}\right)^2}$$