10/22/13
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Solution to Problem 11.14

Bohr Model w/o quantization

$$\left.\begin{aligned} &* \; E = \tfrac{1}{2}mv^2 - \frac{e^2}{4\pi\epsilon_0 r} \\ &* \; \frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2} \end{aligned}\right\} \rightarrow \left\{\begin{aligned} &E = \frac{-e^2}{8\pi\epsilon_0 r} = -\frac{\alpha}{2} mc^2 \frac{\lambda_c}{r} \\ &V = \sqrt{\frac{e^2}{4\pi\epsilon_0 r m}} = \sqrt{\frac{\alpha \lambda_c}{r}}\, c \end{aligned}\right\}$$

where $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c}$ & $\lambda_c = \frac{\hbar}{mc}$

$* \; \text{NB} \; \frac{v}{c} = \sqrt{\frac{\alpha\lambda_c}{r}} << 1$ for $r >> \alpha\lambda_c \cong 2.7 \times 10^{-15}\,m$

$\therefore$ the system is non-relativistic for most of its trajectory

Larmour Formula

$$* \; a = \frac{v^2}{r} = \frac{e^2}{4\pi\epsilon_0 m r^2} = \alpha \frac{\lambda_c c^2}{r^2}$$

$\mu_0 = \frac{1}{\epsilon_0 c^2}$

$$* \; \underline{P} = \frac{\mu_0 e^2 a^2}{6\pi c} = \frac{2}{3} \frac{\alpha^3 \hbar c^2 \lambda_c^2}{r^4}$$

Energy Loss

$$* \; \frac{dE}{dt} = \frac{1}{2}\alpha \frac{mc^2\lambda_c}{r^2}\dot{r} = -\frac{2}{3}\alpha^3 \frac{\hbar c^2 \lambda_c^2}{r^4} \rightarrow -3r^2\dot{r} = 4\alpha^2 c \lambda_c^2$$

$$* \; -3\int_a^0 dr\, r^2 = a^3 = 4\alpha^2 c \lambda_c^2 \Delta t \rightarrow \Delta t = \frac{a^3}{4\alpha^2 \lambda_c^2 c}$$

Plugging in the #'s

$$\left.\begin{aligned} &* \; \alpha \cong \frac{1}{137} \\ &* \; \lambda_c \cong 3.86 \times 10^{-13}\,m \\ &* \; a \cong 5.29 \times 10^{-11}\,m \end{aligned}\right\} \rightarrow \Delta t \cong 1.55 \times 10^{-11}\,s$$