10/29/13
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## Solution to Problem 11.23

(a) * Recall my Oct 23 brilliant & error-free lecture on radiation from a magnetic dipole $\vec{m}(t)$

$$\rightarrow \vec{S} = \frac{\mu_0}{16\pi^2 c^3} \frac{\|\hat{r}\times\ddot{\vec{m}}(t-r/c)\|^2}{r^2}\hat{r} + O\left(\tfrac{1}{r^3}\right) \rightarrow \int d\vec{a}\cdot\vec{S} = \frac{\mu_0 \|\ddot{\vec{m}}\|^2}{6\pi c^3}$$

* if the dipole is $\vec{M}$ at $t=0$ & it rotates at ang. velocity $\vec{\omega}$

$$\rightarrow m^i(t) = \left\{\left[\delta^{ij} - \hat{\omega}^i\hat{\omega}^j\right]\cos(\omega t) + \epsilon^{ijk}\hat{\omega}^k \sin(\omega t) + \hat{\omega}^i\hat{\omega}^j\right\} M^j$$

$$\rightarrow \ddot{m}^i(t) = -\omega^2\left\{\left[\delta^{ij} - \hat{\omega}^i\hat{\omega}^j\right]\cos(\omega t) + \epsilon^{ijk}\hat{\omega}^k\sin(\omega t)\right\} M^j$$

$$\rightarrow \|\ddot{\vec{m}}\|^2 = \omega^4\left\{\left[M^2 - (\hat{\omega}\cdot\vec{M})^2\right]\cos^2(\omega t) + \underbrace{\|\vec{M}\times\hat{\omega}\|^2}_{M^2-(\hat{\omega}\cdot\vec{M})^2}\sin^2(\omega t)\right\} = \omega^4 M^2 \sin^2(\Psi)$$

$$\therefore \boxed{P = \frac{\mu_0 \omega^4 M^2}{6\pi c^3}\sin^2(\Psi)}$$

(b) * Dipole field $\rightarrow \vec{B} = \frac{\mu_0}{4\pi}\frac{[3\hat{r}\hat{r}\cdot\vec{M} - \vec{M}]}{r^3} \xrightarrow{\hat{r}\cdot\vec{M}=0} -\frac{\mu_0 \vec{M}}{4\pi r^3}$

$$* M = \frac{4\pi r^3 B}{\mu_0} = \frac{4\pi (6.4\cdot 10^6 m)^3 (5\times 10^{-5}\frac{kg}{Cs})}{4\pi\times 10^{-7} kg\cdot m/C^2} \simeq \boxed{1.3\times 10^{23}\ Cm^2/s}$$

(c) $$* P \simeq \frac{(4\pi\cdot 10^{-7}\frac{kg\,m}{C^2})(\frac{2\pi}{8.64\cdot 10^4 s})^4 (1.3\times 10^{23}\frac{Cm^2}{s})^2}{6\pi\ (3\times 10^8 m/s)^3}\sin^2(11^\circ) \simeq \boxed{4.3\times 10^{-5}\ W}$$

(d) $$* M = \frac{4\pi R^3 B}{\mu_0} \simeq \frac{4\pi(10^4 m)^3(10^8\frac{kg}{Cs})}{4\pi\times 10^{-7} kg\cdot m/C^2} = \boxed{10^{27}\ Cm^2/s}$$ ← NB not much larger than Earth's

$$* P \simeq \frac{(4\pi\cdot 10^{-7}\frac{kg\cdot m}{C^2})(\frac{2\pi}{10^{-3}s})^4(10^{27} Cm^2/s)^2}{6\pi\ (3\times 10^8 m/s)^3}\sin^2(\Psi) \simeq \boxed{4\times 10^{36}\ W\times\sin^2(\Psi)}$$

* NB if we estimate $\sin^2(\Psi) = 1/2$ we get the Ostriker-Gunn result