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Solution to Problem 12.10

In $\mathcal{O}'$ (Dock) Frame

* Top of mast is $X_T'^{\mu} = (ct', L_4' \hat{n}')$ where $\hat{n}' = \cos\theta' \hat{x}' + \sin\theta' \hat{z}'$

* Bottom of mast is $X_B'^{\mu} = (ct', \vec{0})$

In $\mathcal{O}$ (Dock) Frame $\hat{\beta} = \hat{x}$ ➔

$$\begin{cases} ct = \gamma(ct' + \beta \hat{x}' \cdot \vec{x}') \\ \vec{x} = \vec{x}' - \hat{\beta}\hat{\beta}\cdot\vec{x}' + \gamma\hat{\beta}(\hat{\beta}\cdot\vec{x}' + \beta ct') \end{cases}$$

* Top of mast $\begin{cases} ct = \gamma(ct' + L\vec{\beta}\cdot\hat{n}') \\ \vec{x}_T = L_4'(\hat{n}' - \hat{\beta}\hat{\beta}\cdot\hat{n}') + \gamma\hat{\beta}(L_4'\hat{n}'\cdot\hat{\beta} + \beta ct') \end{cases}$ ➔ $\gamma ct' = ct - \gamma L\vec{\beta}\cdot\hat{n}'$

$$\vec{x}_T = L_4'\left[\hat{n}' - \vec{\beta}\hat{\beta}\cdot\hat{n}' + \frac{1}{\gamma}\vec{\beta}\hat{\beta}\cdot\hat{n}'\right] + \vec{v}t$$

* Bottom of mast $\begin{cases} ct = \gamma ct' \\ \vec{x}_B = \gamma\vec{v}t' = \vec{v}t \end{cases}$

$$\therefore \Delta\vec{x} = \vec{x}_T(t) - \vec{x}_B(t) = L_4'\left[\underbrace{\hat{n}' - \vec{\beta}\hat{\beta}\cdot\hat{n}'}_{\sin\theta'\hat{z}} + \underbrace{\frac{\vec{\beta}\hat{\beta}\cdot\hat{n}'}{\gamma}}_{\frac{1}{\gamma}\cos\theta'\hat{x}}\right]$$

➔ $L_4 \equiv$ length of mast in Dock Frame $= \sqrt{1 - (\vec{\beta}\cdot\hat{n}')^2}\, L_4'$

& $\boxed{\tan\theta = \gamma \tan\theta'}$