11/5/13
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Solution to Problem 12.16

(a) * her total travel time = 18 yrs → 9 yrs each way in her rest frame
* her frame moves at $\pm\frac{4}{5}c$ wrt her brother → $\gamma = 5/3$
∴ her brother is 21 yrs + $\frac{5}{3}$ * 18 yrs = (51 yrs old)

(b) * her brother measures her outward journey as taking 15 yrs
* stay's position is $\Delta X = \frac{4}{5}c * 15\text{ yrs} =$ (12 lyrs)

Moving Twin's Worldline

* $0 < t < 15\text{ yrs} \rightarrow X(t) = \frac{4}{5}ct$
* $15\text{ yrs} < t < 30\text{ yrs} \rightarrow X(t) = 12\text{ lyrs} - \frac{4}{5}c[t - 15\text{ yrs}] = 24\text{ lyrs} - \frac{4}{5}ct$

(c) * she jumps at $(X, t) = (12\text{ lyrs}, 15\text{ yrs})$

(d) * in the $\bar{S}$ frame we have
$$\left\{ \begin{array}{l} c\bar{t} = \frac{5}{3}(15\text{ lyrs} - \frac{4}{5}\cdot 12\text{ lyrs}) = 9\text{ lyrs} \\ \bar{X} = \frac{5}{3}(12\text{ lyrs} - \frac{4}{5}15\text{ lyrs}) = 0 \end{array} \right\}$$
∴ $(\bar{X}, \bar{t}) = (0, 9\text{ yrs})$

(e) * in the $\hat{S}$ frame we have
$$\left\{ \begin{array}{l} c\hat{t} = \frac{5}{3}(15\text{ lyrs} + \frac{4}{5}\cdot 12\text{ lyrs}) = 41\text{ lyrs} \\ \hat{X} = \frac{5}{3}(12\text{ lyrs} + \frac{4}{5}\cdot 15\text{ lyrs}) = 40\text{ lyrs} \end{array} \right\}$$
∴ $(\hat{X}, \hat{t}) = (40\text{ lyrs}, 41\text{ yrs})$

(f) * at the jump her watch reads $\bar{t} = 9$ yrs
* if she resets her watch to $\hat{t} = 41$ yrs she must advance it by 32 yrs
* the trip back takes 9 yrs in the $\hat{S}$ frame → she returns at $\hat{t} = 50$ yrs

(g)

	Brother's Worldline	Position at Jump	Brother's S-time at Jump
$t = 15$ yrs	* in S → $X(t) = 0$	$X = 0$	
$\bar{t} = 9$ yrs	* in $\bar{S}$ → $\bar{X}(\bar{t}) = -\frac{4}{5}c\bar{t}$	$\bar{X} = -\frac{36}{5}$ lyr	$t = \frac{5}{3}(9 + \frac{4}{5}\cdot -\frac{36}{5})$ yrs $= \frac{27}{5}$ yrs
$\hat{t} = 41$ yrs	* in $\hat{S}$ → $\hat{X}(\hat{t}) = +\frac{4}{5}c\hat{t}$	$\hat{X} = +\frac{164}{5}$ lyr	$t = \frac{5}{3}(41 - \frac{4}{5}\cdot\frac{164}{5})$ yrs $= \frac{123}{5}$ yrs

∴ brother's age just before jump = 21 yrs + $\frac{27}{5}$ yrs = (26.4 yrs)
& brother's age just after jump = 21 yrs + $\frac{123}{5}$ yrs = (45.6 yrs)

(h) * the brother ages Δt in his rest frame
* the trip requires $\Delta\hat{t} = 9$ yrs in the $\hat{S}$ frame
} → $\Delta t = \frac{\Delta\hat{t}}{\gamma} =$ ($\frac{27}{5}$ yrs)

∴ the brother's age when his sister returns is 45.6 yrs + 5.4 yrs = (51 yrs)
* of course this is what we got in part (a)