11/9/13
R.P. Woodard

Solution to Problem 12.25

(a) * $\cos(45^\circ) = \sin(45^\circ) = \frac{1}{\sqrt{2}} \rightarrow u_x = u_y = \sqrt{\frac{2}{5}}\, c$

(b) * $1 - u^2/c^2 = 1/5 \rightarrow \frac{1}{\sqrt{1-u^2/c^2}} = \sqrt{5}$

$\therefore \eta_x = \eta_y = \sqrt{2}\, c$

(c) * $\eta^0 = \frac{c}{\sqrt{1-u^2/c^2}} = \sqrt{5}\, c$

* NB $(\eta^0)^2 - \vec{\eta}\cdot\vec{\eta} = c^2$ ☺

(d) * the transformed proper velocity is $\left(\vec{\beta} = \sqrt{\frac{2}{5}}\,\hat{x}\ \&\ \gamma = \sqrt{5/3}\right)$

$\eta^{0\prime} = \gamma(\eta^0 - \beta\eta_x) = \sqrt{3}\, c$

$\eta_x' = \gamma(\eta_x - \beta\eta^0) = 0$

$\eta_y' = \eta_y = \sqrt{2}\, c$

* $\vec{u}' = \frac{\vec{\eta}'}{\eta^{0\prime}}\, c = \sqrt{\frac{2}{3}}\, c\, \hat{y} \rightarrow u_x' = 0\ \&\ u_y' = \sqrt{\frac{2}{3}}\, c$

* the general formula is

$$\vec{u}' = \frac{\hat{\beta}\left[\hat{\beta}\cdot\vec{u} - \vec{v}\right]}{1 - \vec{v}\cdot\vec{u}/c^2} + \frac{\vec{u} - \hat{\beta}\,\hat{\beta}\cdot\vec{u}}{\gamma\left[1 - \vec{v}\cdot\vec{u}/c^2\right]} \xrightarrow[\vec{u} = \sqrt{\frac{2}{5}}\, c(\hat{x}+\hat{y})]{\vec{\beta} = \sqrt{\frac{2}{5}}\,\hat{x}} 0 + \frac{\sqrt{\frac{2}{5}}\, c\,\hat{y}}{\sqrt{\frac{5}{3}}\left[1 - \frac{2}{5}\right]} = \sqrt{\frac{2}{3}}\, c\,\hat{y}$$

(Physics works!)