11/9/13
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Solution to Problem 12.35

* it's just as simple to work this for an arbitrary opening angle so I will

$\boxed{\text{Initial 4-Momenta}}$

* $e^- \to p_-^\mu = (\frac{E}{c}, \vec{p}_e)$ where $E = \sqrt{m^2c^4 + p_e^2c^2}$ (unless the string theorists ...)

* $e^+ \to p_+^\mu = (mc, \vec{0})$

* NB the initial 4-momentum is $p^\mu = p_+^\mu + p_-^\mu$

& $p_\mu p^\mu = 2m(E + mc^2) \neq 0 \to$ the e^+e^- cannot give a single photon

$\boxed{\text{Final 4-Momentum}}$

* $k_1^\mu = \frac{\omega_1}{c}(1, \hat{n}_1) \to k_1^2 = 0$ (photons are massless)

* $k_2^\mu = \frac{\omega_2}{c}(1, \hat{n}_2) \to k_2^2 = 0$

$\boxed{\text{4-Momentum Conservation}}$

* $p_+^\mu + p_-^\mu = k_1^\mu + k_2^\mu \to k_2^\mu = p_+^\mu + p_-^\mu - k_1^\mu$

* $k_2^2 = 0 = (p_+ + p_-)^2 - 2(p_+ + p_-) \cdot k_1 + \cancel{k_1^2} \to 0$

* $(p_+ + p_-)^2 = (\frac{E}{c} + mc)^2 - p_e^2 = \frac{1}{c^2}[m^2c^4 + \cancel{p_e^2c^2}] + 2mE + m^2c^2 - \cancel{p_e^2}$

$= 2m(E + mc^2)$

* $2(p_+ + p_-) \cdot k_1 = 2\frac{\omega_1}{c}\left[\frac{E}{c} + mc - \vec{p}_e \cdot \hat{n}_1\right]$

$$\therefore \boxed{\omega_1 = \frac{mc^2(E + mc^2)}{E + mc^2 - \hat{n}_1 \cdot c\vec{p}_e} \xrightarrow{60^\circ} \frac{mc^2(E + mc^2)}{E + mc^2 - \frac{1}{2}p_ec}}$$