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Solution to Problem 12.44

(a) $\boxed{\text{In the } S \text{ Frame}}$

* q_A is at rest at $\vec{r}_A = 0 \Rightarrow \vec{E}_A(\vec{r}) = \frac{q_A}{4\pi\epsilon_0} \frac{\vec{r}}{\|\vec{r}\|^3}$ & $\vec{B}_A = 0$

* q_B is at $\vec{r}_B(t) = vt\hat{x} + d\hat{y}$

$\therefore \vec{F}_{on B} = q_B \vec{E}_A(\vec{r}_B(t)) = \frac{q_A q_B}{4\pi\epsilon_0} \frac{[vt\hat{x} + d\hat{y}]}{[v^2t^2 + d^2]^3} \xrightarrow[t=0]{} \frac{q_A q_B \hat{y}}{4\pi\epsilon_0 d^2}$

(bi) $\boxed{\text{In the } S' \text{ Frame}}$ (with $\vec{v} = v\hat{x}$ wrt S)

* Recall from class that the force on a particle moving with $\vec{u}$ in S is

$$\vec{F}' = \frac{\vec{F} - \hat{\beta}\hat{\beta}\cdot\vec{F}}{\gamma(1 - \vec{\beta}\cdot\vec{u}/c)} + \frac{\hat{\beta}[\hat{\beta}\cdot\vec{F} - \beta\frac{\vec{u}}{c}\cdot\vec{F}]}{(1 - \vec{\beta}\cdot\vec{u}/c)} \xrightarrow[\vec{u}=\vec{v}]{} \gamma(\vec{F} - \hat{\beta}\hat{\beta}\cdot\vec{F}) + \hat{\beta}\hat{\beta}\cdot\vec{F}$$

* q_A is at $\vec{r}_A'(t') = -vt'\hat{x} \Rightarrow$ it crosses $x' = 0$ at $t' = 0 \Rightarrow t = x = 0$

$\therefore \vec{F} = \frac{q_A q_B \hat{y}}{4\pi\epsilon_0 d^2} \Rightarrow \vec{F}' = \frac{\gamma q_A q_B \hat{y}}{4\pi\epsilon_0 d^2}$

(bii) $\boxed{\text{The } S' \text{ Fields are}}$

* $\vec{E}' = \hat{\beta}\hat{\beta}\cdot\vec{E} + \gamma[\vec{E} - \hat{\beta}\hat{\beta}\cdot\vec{E} + \vec{v}\times\vec{B}] \longrightarrow \frac{\gamma q_A \hat{y}}{4\pi\epsilon_0 d^2}$

* $\vec{B}' = \hat{\beta}\hat{\beta}\cdot\vec{B} + \gamma[\vec{B} - \hat{\beta}\hat{\beta}\cdot\vec{B} - \vec{v}\times\vec{E}/c^2] \xrightarrow{\text{at } t=0} -\frac{\gamma q_A \hat{z}}{4\pi\epsilon_0 c d^2}$

$\therefore \vec{F}' = q_B(\vec{E}' + \vec{0}\times\vec{B}') = \frac{\gamma q_A q_B \hat{y}}{4\pi\epsilon_0 d^2}$