11/9/13
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Solution to Problem 12.50

3+1 Decompose

* $F^{\mu\nu}F_{\mu\nu} = -2F^{0i}F^{0i} + F^{ij}F^{ij}$
* $G^{\mu\nu}G_{\mu\nu} = -2G^{0i}G^{0i} + G^{ij}G^{ij}$
* $F^{\mu\nu}G_{\mu\nu} = -2F^{0i}G^{0i} + F^{ij}G^{ij}$

Substitute Fields & use $\epsilon^{ijk}\epsilon^{ijl} = 2\delta^{kl}$

* $F^{0i} = \frac{1}{c^2}E^i$
* $F^{ij} = \epsilon^{ijk}B^k$
* $G^{0i} = B^i$
* $G^{ij} = -\frac{1}{c^2}\epsilon^{ijk}E^k$

$\rightarrow$

$$F^{\mu\nu}F_{\mu\nu} = -\frac{2}{c^2}\vec{E}\cdot\vec{E} + 2\vec{B}\cdot\vec{B}$$
$$G^{\mu\nu}G_{\mu\nu} = -2\vec{B}\cdot\vec{B} + \frac{2}{c^2}\vec{E}\cdot\vec{E}$$
$$F^{\mu\nu}G_{\mu\nu} = -\frac{4}{c}\vec{E}\cdot\vec{B}$$

* NB $F^{\mu\nu}F_{\mu\nu} = -G^{\mu\nu}G_{\mu\nu}$ is $-\frac{2}{c^2}$ times the 2nd invariant of 12.46
* $F^{\mu\nu}G_{\mu\nu}$ is $-\frac{4}{c}$ times the 1st invariant of problem 12.46