8/12/13
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Solution to Problem 7.20

ⓐ * Recall eqn (5.38) → $\vec{B} \simeq \frac{1}{2}\mu_0 I_{big} \frac{b^2 \hat{z}}{(b^2+z^2)^{3/2}}$

* $\Phi_{small} = \frac{\pi}{2}\mu_0 I_{big} \frac{a^2 b^2}{(b^2+z^2)^{3/2}}$

ⓑ * Recall eqn (5.84) → $\vec{m} = I_{small}\pi a^2 \hat{z}$ → $m = \pi a^2 I_{small}$

* Recall eqn (5.87) → $\vec{B} = \frac{\mu_0}{4\pi} \frac{(3\hat{r}\,\hat{r}\cdot\vec{m} - \vec{m})}{(s^2+z^2)^{3/2}}$

where $\hat{r} = \frac{s\cos\phi\,\hat{x} + s\sin\phi\,\hat{y} - z\hat{z}}{\sqrt{s^2+z^2}}$

* $d\vec{a} = s\,ds\,d\phi\,\hat{z}$ → $\vec{B}\cdot d\vec{a} = +s\,ds\,d\phi * \frac{\mu_0 m}{4\pi}\left[\frac{3z^2}{(s^2+z^2)^{5/2}} - \frac{1}{(s^2+z^2)^{3/2}}\right]$

$\therefore \Phi_{big} = \int d\vec{a}\cdot\vec{B} = \frac{1}{2}\mu_0 m \int_0^b ds\, s\left[\frac{-1}{(s^2+z^2)^{3/2}} + \frac{3z^2}{(s^2+z^2)^{5/2}}\right] = \frac{1}{2}\mu_0 m * \frac{b^2}{(b^2+z^2)^{3/2}}$

→ $\Phi_{big} = \frac{\pi}{2}\mu_0 I_{small} \frac{a^2 b^2}{(b^2+z^2)^{3/2}}$

ⓒ * $\Phi_{small} = \frac{\pi}{2}\mu_0 \frac{a^2 b^2}{(b^2+z^2)^{3/2}} * I_{big}$ → $M_{small\,big} = \frac{\pi}{2}\mu_0 \frac{a^2 b^2}{(b^2+z^2)^{3/2}}$

* $\Phi_{big} = \frac{\pi}{2}\mu_0 \frac{a^2 b^2}{(b^2+z^2)^3} * I_{small}$ → $M_{big\,small} = \frac{\pi}{2}\mu_0 \frac{a^2 b^2}{(b^2+z^2)^{3/2}}$