

Solution to Problem 7.34

$$\vec{E}(\vec{r}, t) = \frac{-q \delta(\vec{r} - \vec{v}t)}{4\pi\epsilon_0 r^2} \quad \& \quad \vec{B}(\vec{r}, t) = 0$$

\* the only eqns we can "check" are the ones without sources

\* Eqn (ii)  $\vec{\nabla} \cdot \vec{B} = 0$   $\left(\infty\right)$

\* Eqn (iii)  $\vec{\nabla} \times \vec{E} = 0$  because  $\vec{E} = -\vec{\nabla}V$  where  $V = \frac{-q}{4\pi\epsilon_0} \left[ \frac{\delta(\vec{r} - \vec{v}t)}{r} + \frac{\delta(\vec{r} - \vec{v}t)}{v} \right]$   
 $= -\frac{\partial \vec{B}}{\partial t}$   $\left(\infty\right)$

\* Eqn (i) really fixes  $\vec{g} = \epsilon_0 \vec{\nabla} \cdot \vec{E} = -q \delta^3(\vec{r}) + \frac{q}{4\pi r^2} \delta(\vec{r} - \vec{v}t)$

\* Eqn (iv) fixes  $\vec{J} = \frac{1}{\mu_0} \vec{\nabla} \times \vec{B} - \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \frac{q v \delta(\vec{r} - \vec{v}t)}{4\pi r^2} \hat{r}$

\* this represents the creation of a <sup>negative</sup> point charge at the origin at  $t=0$  with <sup>positive</sup> spherical shell expanding at speed  $v$