8/25/13
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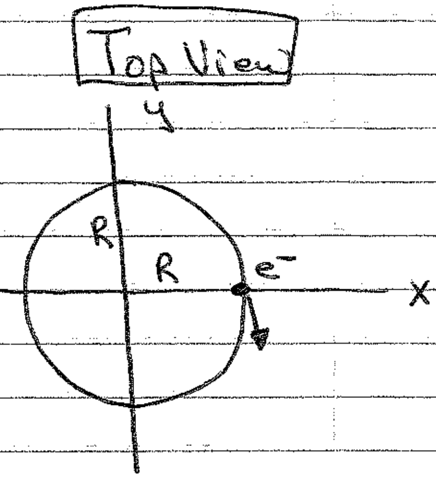
Solution to Problem 7.48

$$\left.\begin{aligned} &* \ \vec{B} = -B(t,s)\hat{z} \\ &* \ d\vec{a} = +\hat{z}\, s\,ds\,d\phi \end{aligned}\right\} \Phi(t) = -2\pi\int_0^R ds\, s\, B(t,s)$$
$$\equiv -\pi R^2 B_{av}(t)$$

$$\left.\begin{aligned} &* \ d\vec{\ell} = R\,d\phi\,\hat{\phi} \\ &* \ \vec{E} = E(t)\hat{\phi} \end{aligned}\right\} \to \mathcal{E} = 2\pi R E(t) = \pi R^2 \dot{B}_{av}$$



$$* \ \vec{v} = R\dot{\phi}\hat{\phi} \to \vec{a} = R\ddot{\phi}\hat{\phi} - R\dot{\phi}^2\hat{s}$$
$$\to m\vec{a} = mR\ddot{\phi}\hat{\phi} - mR\dot{\phi}^2\hat{s}$$

$$* \ \vec{F} = -e\left[E(t)\hat{\phi} + R\dot{\phi}\hat{\phi} \times -B(t,R)\hat{z}\right] = -eE(t)\hat{\phi} + eRB(t,R)\dot{\phi}\hat{s}$$

$$\to \begin{cases} mR\ddot{\phi} = -eE(t) \\ mR\dot{\phi}^2 = -eRB(t,R)\dot{\phi} \to \dot{\phi}(t) = -\frac{e}{m}B(t,R) \to \ddot{\phi} = -\frac{e}{m}\dot{B}(t,R) \end{cases}$$

$$\therefore -eR\dot{B}(t,R) = -eE(t) = -\frac{e}{2}R\dot{B}_{av}(t) \to \boxed{\dot{B}(t,R) = \frac{1}{2}\dot{B}_{av}(t)}$$