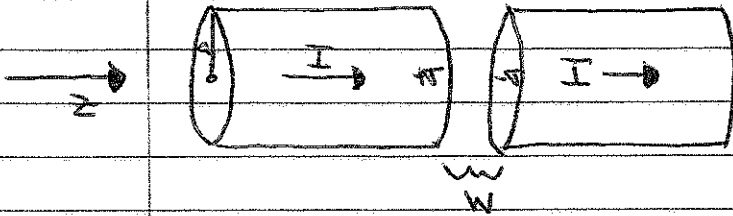


Solution to Problem 8.2

Recall Problem 7.31



$$* Q(t) = \pi a^2 \nabla \cdot \mathbf{H} = I t$$

$$\Rightarrow \nabla \cdot \mathbf{H} = \frac{I t}{\pi a^2}$$

$$* \vec{\nabla} \cdot \mathbf{H} = \frac{\nabla \cdot \mathbf{H}}{\epsilon_0} \hat{z} = \frac{I t}{\pi a^2 \epsilon_0} \hat{z}$$

$$* \vec{\nabla} \cdot \mathbf{E} = \pi a^2 * \frac{I t}{\pi a^2 \epsilon_0} = \frac{I t}{\epsilon_0} \left(\frac{s}{a}\right)^2$$

$$* I_d \equiv \epsilon_0 \frac{\partial \vec{\nabla} \cdot \mathbf{E}}{\partial t} = I \left(\frac{s}{a}\right)^2$$

$$* \vec{B} = B(s) \hat{\phi} \Rightarrow \oint d\vec{\ell} \cdot \vec{B} = 2\pi s B(s)$$

$$* \mu_0 I_d = \mu_0 I \left(\frac{s}{a}\right)^2 \Rightarrow B(s) = \frac{\mu_0 I}{2\pi} \frac{s}{a^2}$$

$$\textcircled{a} \quad \vec{E} = \frac{I t}{\pi a^2 \epsilon_0} \hat{z}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi} \frac{s}{a^2} \hat{\phi}$$

$$\textcircled{b} * u_{em} = \frac{1}{2} \left[\epsilon_0 \vec{E}^2 + \frac{B^2}{\mu_0} \right] = \frac{1}{2\epsilon_0} \left(\frac{I}{\pi a^2} \right)^2 \left[t^2 + \frac{s^2}{4a^2} \right]$$

$$* \vec{\nabla} \cdot \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = -\frac{1}{2\epsilon_0} \left(\frac{I}{\pi a^2} \right)^2 s t \hat{s} \quad \leftarrow \text{directed inward}$$

$$* \text{Eqn 8.14} \Rightarrow \frac{\partial}{\partial t} (u_{med} + u_{em}) = -\vec{\nabla} \cdot \vec{S}$$

$$* \frac{\partial}{\partial t} u_{em} = \frac{1}{2\epsilon_0} \left(\frac{I}{\pi a^2} \right)^2 \cdot 2t$$

$$* -\vec{\nabla} \cdot \vec{S} = \frac{1}{2\epsilon_0} \left(\frac{I}{\pi a^2} \right)^2 * \frac{1}{s} \frac{\partial (s^2 t)}{\partial s} \quad \checkmark \quad (\text{physics works})$$

$$\textcircled{c} * E(t) = \int_0^a ds s \int_0^{2\pi} d\phi \int_0^w dz u_{em} = \frac{1}{2\epsilon_0} \frac{w I^2}{\pi a^2} \left[t^2 + \frac{a^2}{4a^2} \right]$$

$$* - \oint d\vec{a} \cdot \vec{S} = \frac{1}{2\epsilon_0} \frac{I^2}{\pi a^2} \cdot 2\pi a^2 w t = \frac{1}{2\epsilon_0} \frac{w I^2}{\pi a^2} \cdot 2t = \frac{\partial E}{\partial t} \quad \checkmark$$