

# Bose-Einstein Condensation in Cold Atoms: a New State of Matter

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## 1. INTRODUCTION

The timeline of Bose-Einstein condensation nearly covers the past 100 years, from the development of the theory in the 1920s to the creation of a Bose-Einstein condensate in the lab in 1995 and subsequent experiments occurring present-day that utilize these condensates. This development over the years demonstrates clearly the achievement of the impossible through technological progress - when Einstein first developed Bose-Einstein statistics, the cryogenic techniques of the time deemed the creation of BECs impossible, but technological advancements in the 1970s through the 1990s allowed what was once unachievable to be physically observed in the laboratory.

## 2. A BRIEF HISTORY

In 1924 Satyendra Nath Bose devised a statistical mechanical argument in deriving the blackbody spectrum. Unable to publish his paper, Bose sent his paper to Einstein, who not only published the paper but also applied Bose's methods to non-interacting atoms to formulate Bose-Einstein statistics [1, 2]. However, upon its formulation and the subsequent theoretical discovery of Bose-Einstein condensation, Einstein himself considered the physical realization of BEC "folly" and unachievable given the cryogenic techniques and limitations of the time, so experimentalists left the topic largely alone, apart from sparse examples through time [3-5].

Hence, a large gap existed between the initial theoretical derivation of Bose-Einstein condensation and the pursuit of realizing it physically in the laboratory - a span of about 50 years. It wasn't until the 1970s and the advent of laser cooling and magnetic trapping that experimentalists decided that the creation of BECs was possible, given some clever manipulation of the cooling techniques available. Finally, as late as 1995, Wolfgang Ketterle of MIT and two professors from the University of Colorado, Boulder (Eric Cornell and Carl Wieman), separately created a Bose-Einstein condensate in their laboratories - Ketterle developed a BEC using sodium [1] while Cornell and Wieman utilized rubidium [3].

The intellectual pursuit of new phases of matter at ultracold temperatures motivated the developments that led to the creation of Bose-Einstein condensates in Ketterle's and Cornell and Wieman's labs. As Ketterle states, living (theoretically) on the surface of the sun would only reveal the gas phase of atoms; cooling to Earth-like temperatures reveals the solid and liquid phases of matter [1]. As a matter of fact, as recently as

the 20th century, cooling to temperatures on the order of 1 Kelvin revealed superconductivity in 1911 and superfluidity in helium 4 in 1938, while cooling to temperatures on the order of microkelvin revealed superfluidity in helium 3 in 1972 [1].

## 3. BEC THEORY

Bose-Einstein condensation refers to the macroscopic occupation of the atomic ground quantum state in a sample of atoms. Any reference to BEC concerns matter waves - at ultracold temperatures and atomic scales, quantum mechanics dominates physical phenomena. Bose-Einstein condensation actually serves as an example of having the capacity to observe quantum mechanics and wave-particle duality on a macroscopic scale.

The theoretical condition for Bose-Einstein condensation follows: the phase space density must be greater than approximately unity. This translates to the de Broglie wavelength characterizing the wavepacket representing a particular free atom in a sample being sufficiently long enough to overlap with its neighbors. The de Broglie wavelength mentioned in [1] and [3] hold equivalence to the thermal wavelength defined in [6];

$$\lambda_{dB} = \lambda_{th} = \frac{h}{\sqrt{2\pi mk_B T}} \quad (1)$$

The significance of this equivalence lies in the fact that decreasing the temperature of any particular atom (which results from cooling a sample of atoms) increases the de Broglie wavelength of that atom (and subsequently all other atoms in a sample). Therefore, attempts at realizing BEC in the laboratory focused on the cooling of atom samples. Further considerations apply, which the experiment section discusses.

Decreasing temperatures change the behavior of particles from a classical standpoint that involves elastic and inelastic collisions to a quantum mechanical standpoint that involves wave interference and coherence (figure 3). Through minimizing the recombinations of atoms through three-body collisions that eventually form solids at ultracold temperatures while maintaining two-body collisions that decrease the kinetic energy of atoms and produce cooling, Bose-Einstein condensation could be realistically achieved.

The theoretical derivation of Bose-Einstein follows from Blundell and Blundell's derivation [6]. The calculation of the number distribution of particles in a Bose gas is given by

$$N = \int_0^\infty \frac{g(E)dE}{e^{\beta(E-\mu)} - 1}, \quad (2)$$

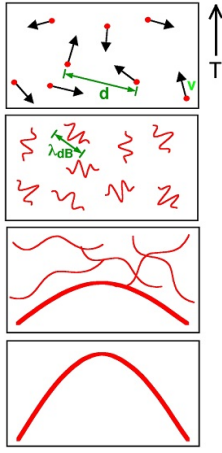


FIG. 1: The behavioral change of atoms as temperatures drop to levels on the order of nanokelvin. Coherence of atomic wavepackets (bottom) leads to the formation of a BEC.[1]

where  $E$  refers to energy,  $\beta = \frac{1}{k_B T}$ , and  $\mu$  refers to the chemical potential. The  $g(E)dE$  term represents the density of states for the Bose gas

$$g(E)dE = \frac{(2S+1)VE^{1/2}dE}{\lambda_{th}^3} \quad (3)$$

where  $V$  is the volume of the sample,  $S$  refers to the spin of the atoms (1 in the case of a boson) and  $\lambda_{th}$  is the thermal wavelength discussed earlier. This gives an equation proportional to the polylogarithm of the fugacity  $z = e^{-\beta\mu}$ :

$$N = \frac{(2S+1)V}{\lambda_{th}^3} Li_{3/2}(z). \quad (4)$$

As the temperature approaches zero, the fugacity term approaches zero as the chemical potential approaches zero (despite the fact that  $\beta$  approaches infinity) until it reaches a temperature  $T_c$ , where the fugacity term equals 1, the polylogarithm becomes the Riemann Zeta function of  $3/2$ , and no solution can be determined for the number distribution. At this point, Bose-Einstein condensation can be theoretically achieved.

Working with the number densities of atoms in the ground state versus atoms in all excited states at temperatures ranging above and below  $T_c$ , it can be determined that the ratio of the population of atoms in the ground state to the total number density of atoms is given by

$$\frac{n_0}{n} = \frac{n - n_1}{n} = 1 - \left(\frac{T}{T_c}\right)^{\frac{3}{2}}. \quad (5)$$

Therefore, as temperatures drop below  $T_c$ , the ratio of the occupation of the ground state by atoms in a sample to the total number density increases to 1 as  $T$  approaches 0.

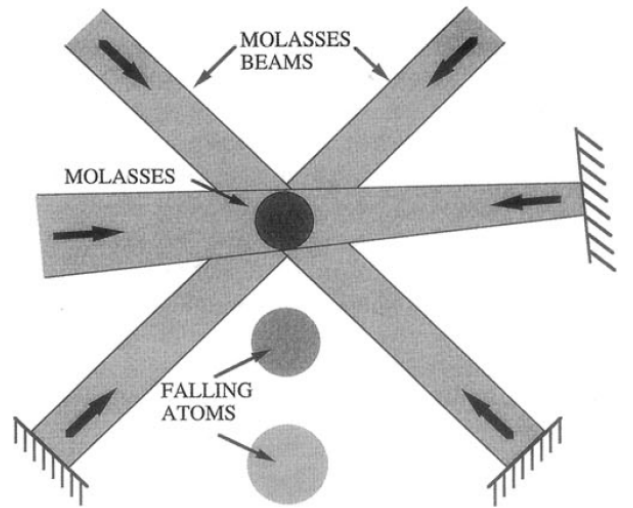


FIG. 2: Illustration of a six beam doppler cooling apparatus. Though the atoms are effectively slowed, they are still subject to the forces of gravity. Figure from [7]

#### 4. THE EXPERIMENTAL HISTORY OF BEC

Though we may think of Einstein's dismissal of Bose-Einstein condensates premature at this point, the history of BEC research has shown that, while not impossible, it's extremely difficult. A BEC is an incredibly fragile system. Even measuring it disrupts the delicate equilibrium. Though  $^4\text{He}$  had been liquified at the beginning of the 20th century, the temperatures for a Bose-Einstein condensate lay six orders of magnitude below that. It was clear in Einstein's time that a major breakthrough in cryogenic techniques would be needed to realize pure BEC in the lab.

The first of many such breakthroughs came in 1968 when Letokhov introduced the doppler cooling technique, followed by further demonstration in the early 70s[3]. In laser cooling techniques such as this, counterpropagating beams of light are used to slow gas phase atoms. In doppler cooling the lasers are tuned to just below the absorption frequency of the atoms in the cloud. Fast atoms moving in a direction towards a light beam see the frequency of the light blue shifted due to the doppler effect. Such atoms have an increased probability of absorbing a photon, which carries momentum opposite to the atom. Photons approaching the atoms from behind are red shifted, and thus are much less likely to be absorbed[7, 8]. With six identical beams, as in Fig. 4, motion in every direction is countered, and the atoms are cooled. With this technique, it is possible to cool down to the  $\mu\text{K}$  scale[1].

As we cool the atoms we wish to thermally isolate them as well. Furthermore, the massive atoms that more easily condense are also more greatly affected by gravity, so we would like to counteract the Earth's pull in our experiment[7, 8]. To accomplish both of these tasks, as

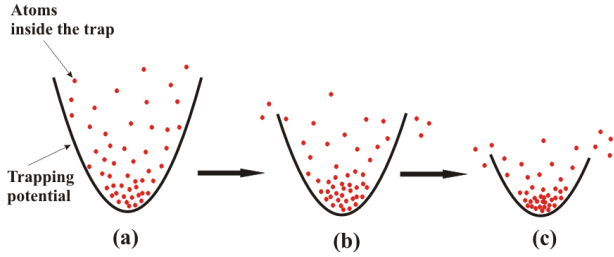


FIG. 3: Illustration of the evaporative cooling technique. High energy atoms are allowed to escape over the sides of the potential well. From [9]

well as to contain the atoms with minimal interference of the laser beams, the magneto-optical trap (MOT) was invented. This brilliantly simple technique swaddles the atoms in a quadrupole magnetic field[3, 7]. By adjusting the coils producing the field, the effects of gravity can be countered while simultaneously giving us a means to dynamically contain the cloud[8]. One flaw in the MOT was that the quadrupole field contained a hole at the center that the low temperature atoms could escape through. The solution to this was to apply an orbiting potential well on top of the static quadrupole, moving fast enough that the atoms could not find the holes to escape through[3].

With the invention of the MOT in the mid 80s[3] the  $\mu\text{K}$  limit had finally been reached. To go any further would require a new technique, as doppler cooling had already reached its theoretical limit[1, 7]. In a sense, the experiments went back to the basics. Evaporation, the original cryogenic technique, turned out to be the answer. With the atoms in a magnetic trap, the magnitude of the potential was decreased, allowing the atoms with the highest energy to escape (see Fig. 4)[1]. This left only the coldest of the cold behind with a temperature well below  $1 \mu\text{K}$ .

All through these 20 years of advances the issue of density remained unresolved. To raise the critical temperature for condensation the density should be high. A high density is also desired for evaporative cooling, so that your final mass is still macroscopic. At the same time, too high of a density and laser cooling breaks down as the three-body collisions begin to outpace the elastic collisions[1, 3]. A balance must be struck that allows both the laser and evaporative portions of the experiment to work.

One solution to this is what came to be known as a Dark Spot. Developed at MIT, this idea called for a shadow to be cast directly down the center of the lasers in the laser cooling setup. This created a spot in the center where cold atoms could spend more time away from collisions with the high energy atoms, allowing the density to be increased[3]. Another solution was simply to adiabatically compress the cold gas with the magnetic field after the evaporative cooling portion[3]. Both techniques increased the allowed densities and therefore the

critical temperature.

The cryogenic breakthroughs pioneered from 1970-1990 within the teams at Boulder and MIT occurred nearly simultaneously. The teams were neck and neck in their development of their apparatus[1]. Their cocreation of a Bose-Einstein condensate in 1995 earned both teams the 2001 Nobel Prize.

## 5. THE ATOM LASER

Recent excursions into research involving Bose-Einstein condensation involve manipulating the quantum mechanical nature of the condensates. One such example is the atom laser, which Ketterle has contributed heavily to. [9] The atom laser holds many analogies to the optical laser, the distinguishing factor between the two being that optical lasers utilize photons while atom lasers utilize matter waves. Thus, the two types of laser share similar properties and applications.

Atom lasers have the capability to be focused to a pinpoint, increasing the intensity of atom radiation over a reduced surface area. They also have the ability to be collimated, which involves many atom waves becoming parallel to one another - this allows an atom laser to travel long distances without spreading[9].

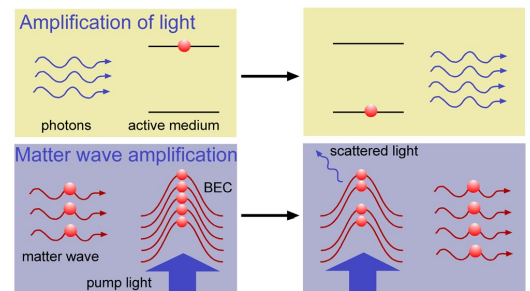


FIG. 4: Schematic for the amplification of an atom laser.

Primary differences in the atom laser from its optical laser sibling lie in the photon/atom distinction. Atoms, unlike photons, cannot be created or destroyed, so amplification of an atom laser (see figure 4) involves pumping more atoms into the ground quantum state through quantum coherence. Matter waves are also affected by gravity, as their masses are large compared to photons (that have zero mass).[9]

Applications involving atom lasers are limited due to their existence solely in vacuum. However, they can be utilized where standard atomic beams are used, such as in atom clocks, atom optics, precision measurements of fundamental constants, and even nanotechnology applications such as atomic beam deposition for computer chip production.

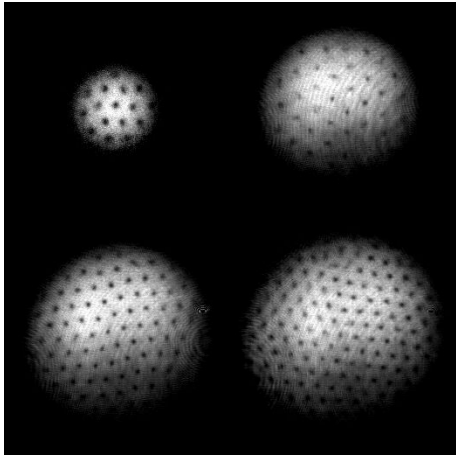


FIG. 5: Images of several vortex lattices in superfluid  $^4\text{He}$ . From [9].

## 6. QUANTUM VORTICES

Another group of experiments involving BECs is used for research into turbulence. Turbulence is one of the least understood areas of the physical world these days, partially because it is extremely complex by its very nature. Turbulence is chaotic and fluctuates rapidly, a combination which does not lend itself well to reproducible experiments. This leaves the theorists with very little empirical evidence with which to develop their theories.

However, as was explained in 1938, superfluid  $^4\text{He}$  is in a state of Bose-Einstein condensation[1], and thus most particles within it have macroscopic wavelengths. One property of a matter wave is that if its constrained to a closed loop, it must necessarily constructively interfere

with itself. Thus the circumference of the path must be an integer multiple of the wavelength of the particle.

If a circular sample of a fluid is set rotating, it's particles will begin to follow circular paths. In superfluid  $^4\text{He}$ , the matter waves must then form closed loops with macroscopic circumference. This leads to a lattice of vortices in the fluid as seen in Fig. 6. Such a simple, easily reproducible system is a huge boon for theorists, as it gives them a system to measure their models against.

## 7. CONCLUSIONS

After many years of hard work and dedication to what was once thought to be a foolish enterprise, Bose-Einstein condensates have been formed out of compounds ranging from rubidium to (spin-polarized) hydrogen[1, 3]. As a technical breakthrough alone this is stupendous, but this has also opened the doors to totally new things. Atomic lasers can be used in a myriad of applications where they significantly increase the effectiveness. Quantum vortices give us insight into an otherwise nigh untouchable region of the physical world. In short: the impossible is now possible.

## 8. ACKNOWLEDGMENTS

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