Chap. 10 Hmwk

10.2 (a) $D(k) = \frac{V}{2\pi^2} k^2 dk$ $\qquad \frac{(\hbar k)^2}{2m} = E$

$$k = \frac{\sqrt{2mE}}{\hbar}$$

$$D(E) = \frac{V}{2\pi^2} \cdot \left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2} dE \qquad dk = \frac{1}{2}\sqrt{\frac{2m}{\hbar^2}}\, E^{-1/2} dE$$

$T=0$ all states up to E_F occupied
$f(E)=1$

$$\langle U \rangle = \int_0^{E_F} E\, D(E)\, dE$$

$$= \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \frac{2}{5} E_F^{5/2}$$

$$= \underline{\frac{3}{5} E_F N} \quad \text{using} \quad N = \frac{V}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E_F^{3/2}$$

$$E_F = \left(\frac{3\pi^2}{VN}\right)^{2/3} \cdot \left(\frac{\hbar^2}{2m}\right)$$

(b) ∴ Can write

$$\langle U \rangle = \frac{3}{5}\left(\frac{\hbar^2}{2m}\right)\left(\frac{3\pi^2}{V}\right)^{2/3} N^{5/3}$$

$$P = -\frac{\partial U}{\partial V} = +\frac{2}{5}\left(\frac{\hbar^2}{2m}\right)(3\pi^2)^{2/3} V^{-5/3} N^{5/3}$$

$$= \frac{2}{5}\frac{N}{V} E_F$$

$\longrightarrow$

(c) $P = \frac{2}{5} n E_F$

$m_n = 1.67 \times 10^{-27}$ kg.

$n = \frac{\rho}{m_n} = \frac{10^{18}}{1.67 \times 10^{-27}} = 6.10^{44} \ m^{-3}$.

$k_F = (3\pi^2 n)^{1/3} = 2.6 \times 10^{15} \ m^{-1}$

$E_F = \frac{\hbar^2 k_F^2}{2m} = 2.3 \times 10^{-11}$ J.

$P = \frac{2}{5} n E_F = 5.5 \times 10^{33}$ Pa.

3. No protons $N_p = \frac{3 \times 10^{30}}{1.67 \times 10^{-27}} = 1.8 \times 10^{57} / m^3$

$r = 3 \times 10^7$

$V = \frac{4}{3}\pi r^3 = 4\pi \times 9 \times 10^{21} = 1.1 \times 10^{23}$.

$\frac{N_p}{V} = 1.6 \times 10^{34}$

$k_F = (3\pi^2 n)^{1/3} = 8 \times 10^{11} \ m^{-1}$

$E_F = \frac{\hbar^2 k_F^2}{2m_p} = 2 \times 10^{-18}$ J

$T_F = 1.5 \times 10^5$ K. $< 10^7 = T_{sta}$

8. $T=0$

$$N_{\uparrow} = \frac{1}{2}\int_0^{E_F} dE\, \mathcal{D}(E) + \frac{1}{2}\mu_B B\, \mathcal{D}(E_F)$$

$$N_{\downarrow} = \frac{1}{2}\int \text{- - -} - \frac{1}{2}\mu_B B\, \mathcal{D}(E_F)$$

$$M = \mu_B (N_{\uparrow} - N_{\downarrow}) = \mu_B^2 B (\mathcal{D}(E_F))$$

$$\chi = \frac{1}{V}\frac{\partial M}{\partial H} = \mu_0 \mu_B^2 \frac{\mathcal{D}(E_F)}{V}$$

$$\frac{\mathcal{D}}{V} = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E_F^{1/2}$$

$m = 9\times 10^{-31}$

$\hbar = 1.1\times 10^{-34}$

$$\frac{2m}{\hbar^2} = 1.5\times 10^{38}$$

$E_F = 7\,\text{eV} = 1.1\times 10^{-18}\ \text{J}$

$$\frac{\mathcal{D}}{V} = \frac{1}{2\pi^2}(15)^{3/2}\ 110^{1/2} \cdot 10^{-9} \simeq 3\times 10^{47}$$

$$\chi = 4\pi\cdot 10^{-7}\cdot (9\times 10^{-24})^2 \times 3\times 10^{47}$$

$$= \underline{1.5\times 10^{-5}}$$

10.14 $$P = \frac{2}{5}\frac{N}{V}E_F$$

$$= \frac{2}{5}\frac{N}{V}\frac{\hbar^2}{2m}\left(3\pi^2\frac{N}{V}\right)^{2/3}$$

$$= \left(3\pi^2\right)^{2/3}\frac{\hbar^2}{5m}\left(\frac{N}{V}\right)^{5/3}$$

$$B = -V\left(\frac{\partial P}{\partial V}\right)$$

$$= \left(3\pi^2\right)^{2/3}\frac{\hbar^2}{3m}\left(\frac{N^{5/3}}{V^{5/3}}\right)$$

$$= \frac{2}{3}nE_F$$

K	B = 1.25 GPa
Cs	0.7 "
Cu	64 "

HMWK Chap. 11

1. $$\frac{dP}{dT} = -\frac{L_{GL}}{T_p(V_G - V_L)}$$ (Clapeyron eqn)

$V_G \gg V_L$

$$\frac{dP}{dT} = -\frac{L_{GL}}{T_p V_G}$$ $PV_G = nRT$ gas law

$$\frac{dP}{dT} = -\frac{L_{GL}\,P}{nRT^2}$$

$$\frac{dP}{P} = -\frac{L_{GL}}{nR}\frac{dT}{T^2} = \frac{L_{GL}}{nR}\,d\left(\frac{1}{T}\right)$$

$$P(T) = P(0)\,e^{\left(\frac{L}{nRT}\right)}$$

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2. $$dG = -SdT + MdB$$ assume $\vec{M} \parallel \vec{B}$.

$$G_1(T+dT,\ B+dB) = G_2(T+dT,\ B+dB)$$ equilibrium

Expand in terms of dT + dB

$$\left[\underbrace{\left(\frac{\partial G_1}{\partial B}\right)_T}_{M_1} - \underbrace{\left(\frac{\partial G_2}{\partial B}\right)_T}_{M_2}\right] dB = \left[\underbrace{\left(\frac{\partial G_2}{\partial T}\right)_B}_{S_2} - \underbrace{\left(\frac{\partial G_1}{\partial T}\right)_B}_{S_1}\right] dT$$

2 contd. $$\frac{dB}{dT} = \frac{S_1 - S_2}{M_1 - M_2}.$$ Magnetic Clapeyron

$$= \frac{L_{12}}{T_c} \frac{1}{\Delta M}$$

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4. $$P(z) = P(0) e^{-mgz/k_BT}$$

$$\frac{dP(z)}{dz} = -\left(\frac{mg}{k_BT}\right) P(z).$$ $$\frac{dT_b}{dP} = +\frac{T_b \Delta V}{L_{LG}}.$$ (Clapeyron Eqn)

$$\frac{dT_b}{dz} = \frac{\partial T_b}{\partial P(z)} \frac{dP(z)}{dz}$$

$$= \frac{-mg P(z)}{k_BT} \frac{T_b \Delta V}{L_{LG}}$$

$$= -\frac{mg P(z)}{k_BT} \frac{T_b}{L_{LG}} \left(\frac{\rho_L - \rho_G}{\rho_L \rho_G}\right)$$

Now $$P = \frac{n k_B T}{V}$$ gas law.

$$n = \frac{m}{M}$$ M = molecular mass.

4 contd.

$$P = \frac{\rho}{M} k_B T$$

$$\therefore \frac{dT_b}{dz} = \frac{T_b g}{L_{LG}} \left(\frac{\rho_L - \rho_G}{\rho_L \rho_G} \right)$$

$$= 3 \times 10^{-3} \ \mathrm{K/m}$$

5. For phase separation, the free energy of the mixture,

$$F = \tfrac{1}{2} N z \left[x \epsilon_{AA} + (1-x) \epsilon_{BB} \right] - N z v \, x(1-x)$$

$$+ \underbrace{N k_B T \left[x \ln x + (1-x) \ln(1-x) \right]}_{\text{entropy term}}$$

$$\frac{\partial F}{\partial x} = \tfrac{1}{2} N z \left[\epsilon_{AA} - \epsilon_{BB} \right] - N z v \left[1 - 2x \right] + N k_B T \left[\ln\left(\frac{x}{1-x} \right) \right]$$

$$\frac{\partial^2 F}{\partial x^2} = -2 N z v + N k_B T \left[\frac{1}{x} + \frac{1}{1-x} \right]$$

Condition $\frac{\partial^2 F}{\partial \alpha^2} = 0$ for stable point

i.e. $\frac{1}{x} + \frac{1}{1-x} = \frac{-2zv}{k_B T}$

or $x(1-x) = \frac{k_B T}{2z|v|}$

Maximum value of α is $\alpha = \frac{1}{2}$ for

which $k_B T_{max} = \frac{z|v|}{2}$

Homework Chap. 12

2. Grand potential

$$\Phi_G = \tfrac{1}{2}a\varphi^2 + \tfrac{1}{6}b\varphi^6 - \eta\varphi$$

$a = \alpha(T-T_c)$

$$\frac{\partial\Phi}{\partial\eta} = -\varphi$$

$$\frac{\partial\Phi}{\partial\varphi} = \varphi\left[a + b\varphi^4\right] - \eta$$

$$\chi^{-1} = \frac{\partial}{\partial\varphi}\eta = \frac{\partial\eta}{\partial\Phi}\frac{\partial\Phi}{\partial\varphi} = -\frac{1}{\varphi}\left\{\varphi(a+b\varphi^4) - \eta\right\}$$

a scales as $(T-T_c)^1$

$\chi^{-1} = -a + \cdots$

$\therefore$ χ scales as $\dfrac{1}{T-T_c}$ $\quad\boxed{\gamma = 1}$

At T_c, $a = 0$, $\dfrac{\partial\Phi}{\partial\varphi} = 0$: $\therefore$ η scales as $b\varphi^5$

or for $\eta \sim b\varphi^\delta$ $\quad\therefore \boxed{\delta = 5}$

Term $a + b\varphi^4$ must scale as $(T-T_c)^1$,

hence φ scales as $(T-T_c)^{1/4}$

$$\boxed{\beta = \tfrac{1}{4}}$$

3. See appendix F.

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6. $F = \frac{1}{2}\alpha(T-T_c)P^2 + \frac{1}{4}bP^4 + \frac{1}{6}cP^6 + DxP^2 + \frac{1}{2}Ex^2$

$\left.\frac{\partial F}{\partial x}\right|_P = 0$ Hence, $x = -\frac{D}{E}P^2$

Substitute x back into F, to obtain

$F = \frac{1}{2}\alpha(T-T_c)P^2 + \frac{1}{4}\left(b - \frac{2D^2}{E}\right)P^4 + \frac{1}{6}cP^6$

If coefficient of P^4 term is negative the $P=0$ solution to $F'=0$ is unstable and if $b < \frac{2D^2}{E}$ obtain a 1st order phase transition

7. $\Phi(\varphi_1, \varphi_2) = \frac{1}{2}a_1\varphi_1^2 + \frac{1}{2}a_2\varphi_2^2 + \frac{1}{4}b_{11}\varphi_1^4 + \frac{1}{4}b_{22}\varphi_2^4$
$+ \frac{1}{2}b_{12}\varphi_1^2\varphi_2^2$

Put $\varphi_1 = 0$ and find solution for which Φ has a minimum as function of φ_2

Find $\varphi_2^2 = -a_2/b_{22}$

Substitute this solution back into F

$$\Phi = \frac{1}{2}\left[a_1 - \frac{a_2 b_{12}}{b_{22}}\right]\varphi^2 + \frac{1}{4}b_{11}\varphi^4 + \dots \left\{\text{for } \varphi_1 \text{ small}\right.$$

$$\frac{\partial}{\partial\varphi_1}(\Phi) = \varphi\left\{\left(a_1 - \frac{a_2 b_{12}}{b_{22}}\right) + b_{11}\varphi_1^2\right\}.$$

Apply usual Landau arguments

For stability need $a_1 - \frac{a_2 b_2}{b_{22}} > 0$

or $|a_1| < |a_2| \frac{b_2}{b_{22}}$ (sign of a_1, a_2)

For instability,

$$b_{12} < \frac{b_{22}|a_1|}{|a_2|}$$

For φ_2 small, redo for $\frac{\partial\Phi}{\partial\varphi_2}$ + find $b_{12} < \frac{b_{11}|a_2|}{|a_1|}$