Quiz 1

Let $\psi(x) = Cx^p$ with $p = 2$ on the interval $0 \leq x \leq 1$.

1. What is C so that ψ is normalized on the interval $0 \leq x \leq 1$?

$$1 = \int_0^1 |\psi(x)|^2 dx = \int_0^1 C^2 x^{2p} dx = \frac{C^2}{2p+1} \rightarrow C = \sqrt{2p+1}$$

$$= \sqrt{5} \quad p=2$$
$$\sqrt{7} \quad p=3$$
$$\sqrt{9} \quad p=4$$

2. What is the expectation value of x: $\langle x \rangle$?

$$\langle x \rangle = \int_0^1 x |\psi(x)|^2 dx = \int_0^1 x \, C^2 x^{2p} dx = \frac{2p+1}{2p+2} = \frac{5}{6} \quad p=2$$
$$\frac{7}{8} \quad p=3$$
$$\frac{9}{10} \quad p=4$$

3. What is the expectation value of x^2: $\langle x^2 \rangle$?

$$\langle x^2 \rangle = \int_0^1 x^2 |\psi(x)|^2 dx = \frac{2p+1}{2p+3} = \frac{5}{7} \quad p=2$$
$$\frac{7}{9} \quad p=3$$
$$\frac{9}{11} \quad p=4$$

$$= \langle x^2 \rangle - \langle x \rangle^2$$

4. What is σ_x? $\quad \sigma_x^2 = 2p+1 \left(\frac{1}{2p+3} - \frac{2p+1}{(2p+2)^2} \right)$

$$\rightarrow \sigma_x = \sqrt{\frac{5}{7} - \left(\frac{5}{6}\right)^2}$$

$$= \sqrt{\frac{7}{9} - \left(\frac{7}{8}\right)^2}$$

$$= \sqrt{\frac{9}{11} - \left(\frac{9}{10}\right)^2}$$