Name:

Quiz 2

In the following let $\psi_n(x)$ be the eigenstates of an infinite square well on the interval, $0 < x < 1$:

$$\psi_n(x) = \sqrt{2}\sin(n\pi x). \tag{1}$$

At $t = 0$ the wave function, $\psi(x,0)$, is $1/\sqrt{\Delta}$ for $0 < x < \Delta$ and zero for $\Delta < x < 1$. This wave function can be expressed as a linear combination of the infinite square well eigenstates:

$$\Psi(x,0) = \sum_{n=0}^{\infty} c_n \psi_n(x). \tag{2}$$

1. Calculate c_n.

$$C_n = \int_0^1 dx\, \psi_n^*(x)\, \Psi(x,0)$$

$$= \int_0^{\Delta} dx\, \sqrt{2}\sin(n\pi x)\frac{1}{\sqrt{\Delta}}$$

$$= -\frac{\sqrt{2}}{\sqrt{\Delta}} \frac{\cos(n\pi x)}{n\pi}\Big|_0^{\Delta} = \boxed{\sqrt{\frac{2}{\Delta}}\frac{1}{n\pi}(1-\cos(n\pi\Delta)) = C_n}$$

2. What is the condition on the c_n that the wave function be normalized? (You can just write this down.)

$$1 = \sum_n |C_n|^2$$

3. Write down an expression for the wave function as a function of time, $\Psi(x,t)$.

$$\Psi(x,t) = \sum_{n=1}^{\infty} C_n \psi_n(x) e^{-iE_n t/\hbar}$$

where $E_n = \frac{\hbar^2}{2m}(n\pi)^2$.