Name:

Quiz 3

For the harmonic oscillator hamiltonian, $H = \hbar\omega(a_+a_- + \frac{1}{2})$, the position operator may be written as

$$x = \sqrt{\frac{\hbar}{2m\omega}}\,(a_+ + a_-)\,. \tag{1}$$

At $t = 0$ the wave function is given by

$$\psi(x,0) = \frac{1}{\sqrt{2}}\,(\psi_1(x) + i\psi_3(x))\,. \tag{2}$$

1. What is the wave function at time t: $\psi(x,t)$?

$$\psi(x,t) = \frac{1}{\sqrt{2}}\left(\psi_1(x)\,e^{-i\frac{3}{2}\omega t} + i\psi_3(x)\,e^{-i\frac{7}{2}\omega t}\right)$$

2. What is the expectation value of x as a function of time?

$\langle x\rangle = 0$ because $(a_+ + a_-)$ does not couple ψ_1 & ψ_3 or ψ_n to ψ_n.

3. What is the expectation value of x^2 as a function of time?

$$(a_+ + a_-)^2\psi_1 = \sqrt{6}\,\psi_3 + 3\,\psi_1$$
$$(a_+ + a_-)^2\psi_3 = \sqrt{6}\,\psi_1 + 7\,\psi_3 + \psi_5\ \text{term}$$
$$\Rightarrow \langle x^2\rangle = \frac{\hbar}{2m\omega}\,\frac{1}{2}\left(3 + 7 + i\sqrt{6}\,e^{-i2\omega t} - i\sqrt{6}\,e^{i2\omega t}\right)$$
$$= \frac{\hbar}{2m\omega}\left(5 + \sqrt{6}\sin(2\omega t)\right)$$