Name:

Quiz 5

The Hamiltonian of a two state system has eigenvalues and eigenvectors of

$$E_a = 1 \text{ with } |\psi_a\rangle = 0.6|1\rangle + 0.8i|2\rangle \quad (1)$$
$$E_b = -1 \text{ with } |\psi_b\rangle = 0.8|1\rangle - 0.6i|2\rangle. \quad (2)$$

1. What is $|\psi_a\rangle\langle\psi_a|$? You may express your answer either in terms of a matrix or the Dirac bra-ket notation.

$$|\psi_a\rangle = \begin{pmatrix} 0.6 \\ 0.8i \end{pmatrix} \quad \langle\psi_a| = \begin{pmatrix} 0.6 & -0.8i \end{pmatrix}$$

$$|\psi_a\rangle\langle\psi_a| = \begin{pmatrix} 0.6 \\ 0.8i \end{pmatrix}\begin{pmatrix} 0.6 & -0.8i \end{pmatrix} = \begin{pmatrix} 0.6\cdot 0.6 & 0.6\cdot -0.8i \\ 0.8i\cdot 0.6 & 0.8i\cdot -0.8i \end{pmatrix}$$

$$= \begin{pmatrix} 0.36 & -0.48i \\ 0.48i & 0.64 \end{pmatrix}$$

2. What is $|\psi_b\rangle\langle\psi_b|$? You may express your answer either in terms of a matrix or the Dirac bra-ket notation.

$$|\psi_b\rangle = \begin{pmatrix} 0.8 \\ -0.6i \end{pmatrix} \quad \langle\psi_b| = \begin{pmatrix} 0.8 & 0.6i \end{pmatrix}$$

$$|\psi_b\rangle\langle\psi_b| = \begin{pmatrix} 0.8 \\ -0.6i \end{pmatrix}\begin{pmatrix} 0.8 & 0.6i \end{pmatrix} = \begin{pmatrix} 0.8\cdot 0.8 & 0.8\cdot 0.6i \\ -0.6i\cdot 0.8 & -0.6i\cdot 0.6i \end{pmatrix}$$

$$= \begin{pmatrix} 0.64 & 0.48i \\ -0.48i & 0.36 \end{pmatrix}$$

3. What is the Hamiltonian?

$$H = E_a|\psi_a\rangle\langle\psi_a| + E_b|\psi_b\rangle\langle\psi_b|$$

$$= \begin{pmatrix} -0.28 & -0.96i \\ 0.96i & 0.28 \end{pmatrix}$$