

Name:

Quiz 3

At $t = 0$ a harmonic oscillator wave function is equal to

$$\psi(x, 0) = \frac{1}{\sqrt{2}}(\psi_0(x) - i\psi_2(x)). \quad (1)$$

Using

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-) \quad (2)$$

$$p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-), \quad (3)$$

compute

1. $\psi(x, t)$,

$$\psi(x, t) = \frac{1}{\sqrt{2}} (\psi_0(x) e^{-i\frac{\omega}{2}t} - i\psi_2(x) e^{-i\frac{5\omega}{2}t})$$

2. the expectation value of x as a function of time,

0 because a_{\pm} do not connect ψ_0 & ψ_2 .

3. the expectation value of p as a function of time,

0 because a_{\pm} do not connect ψ_0 & ψ_2 .

4. the expectation value of x^2 as a function of time,

$$\langle x^2 \rangle = \frac{\hbar}{2m\omega} \frac{1}{2} \int dx (\psi_0^* e^{i\frac{\omega}{2}t} + i\psi_2^* e^{i\frac{5\omega}{2}t}) \underbrace{(a_+ + a_-)^2}_{a_+ a_- + a_- a_+ + a_+^2 + a_-^2} (\psi_0 e^{-i\frac{\omega}{2}t} - i\psi_2 e^{-i\frac{5\omega}{2}t}) \rightarrow$$

5. and the expectation value of p^2 as a function of time.

$$\langle p^2 \rangle = \frac{\hbar m\omega}{2} \frac{1}{2} \int dx (\psi_0^* e^{i\frac{\omega}{2}t} + i\psi_2^* e^{i\frac{5\omega}{2}t}) (a_+ a_- + a_- a_+ - a_+^2 - a_-^2) \underbrace{(\psi_0 e^{-i\frac{\omega}{2}t} - i\psi_2 e^{-i\frac{5\omega}{2}t})}_{\rightarrow}$$

$$a_+ a_- \Psi_n = n \Psi_n$$

$$a_- a_+ \Psi_n = (n+1) \Psi_n$$

$$(a_+ a_- + a_- a_+) \Psi_n = (2n+1) \Psi_n$$

$$a_+^2 \Psi_0 = \sqrt{2} \Psi_2$$

$$a_-^2 \Psi_2 = \sqrt{2} \Psi_0$$

$$\rightarrow \langle x^2 \rangle = \frac{\hbar}{2m\omega} \frac{1}{2} \left((2 \cdot 0 + 1) + (2 \cdot 2 + 1) + i e^{2i\omega t} \sqrt{2} - i e^{-2i\omega t} \sqrt{2} \right)$$

$$\boxed{\langle x^2 \rangle = \frac{\hbar}{2m\omega} (3 - \sqrt{2} \sin(2\omega t))}$$

$$\rightarrow \langle p^2 \rangle = \frac{\hbar m \omega}{2} \frac{1}{2} \left((2 \cdot 0 + 1) + (2 \cdot 2 + 1) - i e^{2i\omega t} \sqrt{2} + i e^{-2i\omega t} \right)$$

$$\boxed{\langle p^2 \rangle = \frac{\hbar \omega m}{2} (3 + \sqrt{2} \sin(2\omega t))}$$