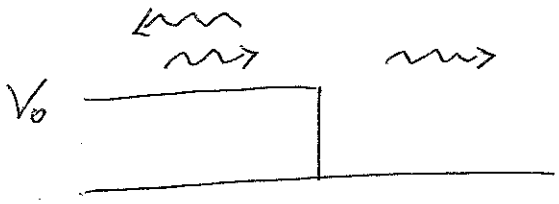
Name:

Quiz 4

In one dimension a particle moves in a potential $V(x) = V_o > 0$ for $x < 0$, and $V(x) = 0$ for $x > 0$.

1. Write down the wave function for a particle incident from the left with $E > V_o$. Make sure to specify the wave vectors in terms of the energy.



$$k_1 = \sqrt{2m(E - V_o)/\hbar^2}$$

$$k_2 = \sqrt{2mE/\hbar^2}$$

$$\psi(x<0) = Ae^{ik_1x} + A'e^{-ik_1x}$$

$$\psi(x>0) = Be^{ik_2x}$$

2. Write down the boundary conditions at $x = 0$.

$$\psi(0) = A + A' = B$$

$$\psi'(0) = ik_1(A - A') = ik_2B \longrightarrow A - A' = \frac{k_2}{k_1}B$$

3. Solve for the transmission probability.

$$2A = \left(1 + \frac{k_2}{k_1}\right)B$$

$$\rightarrow T = \frac{k_2|B|^2}{k_1|A|^2} = \frac{k_2}{k_1}\left(\frac{2}{1 + \frac{k_2}{k_1}}\right)^2 = \boxed{\frac{4k_1k_2}{(k_1+k_2)^2}}$$

check: $$R = \frac{|A'|^2}{|A|^2}\frac{k_1}{k_1} = \left(\frac{1 - k_2/k_1}{1 + \frac{k_2}{k_1}}\right)^2 = \frac{(k_1-k_2)^2}{(k_1+k_2)^2}$$

$$\longrightarrow R + T = 1$$