Name:

Quiz 6

Three Hermitian operators are defined below in a two dimensional space, and their eigenvalues and eigenvectors are given.

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ with } +\frac{\hbar}{2} : \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ and } -\frac{\hbar}{2} : \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
$$S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \text{ with } +\frac{\hbar}{2} : \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} \text{ and } -\frac{\hbar}{2} : \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$
$$S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ with } +\frac{\hbar}{2} : \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } -\frac{\hbar}{2} : \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Initially the system is in the state

$$|\psi\rangle = \begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix}.$$

1. A measurement of S_x is made. What are the possible outcomes and the probabilities of those outcomes?

$+\hbar/2$ with prob. $= \left| \frac{1}{\sqrt{2}} (1 \ \ 1) \begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix} \right|^2 = \frac{1.4^2}{2} = 0.98$

$-\hbar/2$ with prob. $= \left| \frac{1}{\sqrt{2}} (1 \ -1) \begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix} \right|^2 = \frac{0.2^2}{2} = 0.02$

2. The result of the S_x measurement is $-\hbar/2$. What is the state of the system immediately after this measurement?

$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

3. Assuming the system is in the state of part 2, a S_y measurement is made. What are the possible outcomes and the probabilities of those outcomes?

$+\hbar/2$ with prob. $= \left| \frac{1}{\sqrt{2}} (1 \ -i) \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right|^2 = \frac{|1+i|^2}{4} = \frac{1}{2}$

$-\hbar/2$ with prob. $= \left| \frac{1}{\sqrt{2}} (1 \ \ i) \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right|^2 = \frac{|1-i|^2}{4} = \frac{1}{2}$