Solution Name:

Quiz 1

For the following you will need to know the area under a Gaussian,

$$\int_{-\infty}^{\infty} e^{-ax^2}\,dx = \frac{\sqrt{\pi}}{\sqrt{a}}, \tag{1}$$

as well as the related integrals with an additional x^2, etc. which are done by taking derivatives with respect to a.

Consider the wave function $\psi(x) = C(1+ix)e^{-x^2}$.

1. What is the constant, C, so that the wave function is normalized?

$$1 = \int_{-\infty}^{+\infty} dx\, C^2(1-ix)e^{-x^2}(1+ix)e^{-x^2} = C^2\int_{-\infty}^{+\infty} dx\,(1+x^2)e^{-2x^2}$$

$$= C^2\left(\sqrt{\frac{\pi}{2}} + \frac{1}{2}\frac{\sqrt{\pi}}{2^{3/2}}\right) = C^2\sqrt{\frac{\pi}{2}}\left(1+\frac{1}{4}\right) = C^2\frac{5}{4}\sqrt{\frac{\pi}{2}} \rightarrow \boxed{C = \sqrt{\frac{4}{5}}\left(\frac{2}{\pi}\right)^{1/4}}$$

2. What is the expectation value of x for this wave function?

$$\langle x\rangle = \int_{-\infty}^{+\infty} dx\, C^2(1-ix)x(1+ix)e^{-2x^2} = \boxed{0 = \langle x\rangle}$$

3. what is the expectation value of p for this wave function?

$$\langle p\rangle = \int_{-\infty}^{+\infty} dx\, C^2(1-ix)e^{-x^2}\frac{\hbar}{i}\frac{d}{dx}(1+ix)e^{-x^2}$$

$$= C^2\frac{\hbar}{i}\int dx\,(1-ix)e^{-x^2}(i-2x-2ix^2)e^{-x^2}$$

$$= C^2\hbar\int dx\,(1+2x^2-2x^2)e^{-x^2} = \frac{4}{5}\sqrt{\frac{2}{\pi}}\,\hbar\sqrt{\frac{\pi}{2}} = \boxed{\frac{4}{5}\hbar = \langle p\rangle}$$