Solution Name:

Quiz 3

The position and momentum operators expressed in terms of the harmonic oscillator raising and lowering operators are

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-) \tag{1}$$

$$p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-). \tag{2}$$

In this problem consider the harmonic oscillator wavefunction

$$\psi = \frac{1}{\sqrt{2}}(\psi_1 + i\psi_2). \tag{3}$$

1. What is the expectation value of x?

$$\langle x\rangle = \int \frac{1}{\sqrt{2}}(\psi_1^* - i\psi_2^*)\sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-)\frac{1}{\sqrt{2}}(\psi_1 + i\psi_2)\,dx$$

$$= \frac{1}{2}\sqrt{\frac{\hbar}{2m\omega}}\left(-i\sqrt{2} + i\sqrt{2}\right) = 0$$

2. What is the expectation value of p?

$$\langle p\rangle = \int \frac{1}{\sqrt{2}}(\psi_1^* - i\psi_2^*)\, i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-)\frac{1}{\sqrt{2}}(\psi_1 + i\psi_2)\,dx$$

$$= \frac{1}{2}\sqrt{\frac{\hbar m\omega}{2}}\, i\left(-i\sqrt{2} - i\sqrt{2}\right) = \sqrt{\hbar m\omega}$$

3. What is the expectation value of x^2?

$$\langle x^2\rangle = \frac{\hbar}{2m\omega}\frac{1}{2}\int(\psi_1^* - i\psi_2^*)\underbrace{(a_+ + a_-)^2}_{(a_+)^2 + (a_-)^2 + \underbrace{a_-a_+ + a_+a_-}_{\text{yields } 2n+1}}(\psi_1 + i\psi_2)\,dx$$

$$= \frac{\hbar}{2m\omega}\frac{1}{2}\left((2\cdot 1 + 1) + (2\cdot 2 + 1)\right)$$

$$= \frac{2\hbar}{m\omega}$$