Name:

Quiz 4

Consider the one dimensional Schrodinger equation with $\psi(x) = 0$ for $x \leq 0$ because, for example, $V(x) = \infty$ for $x \leq 0$, *and* for $x > 0$ the Schrodinger equation is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} - \frac{\alpha}{x}\psi = E\psi. \tag{1}$$

This problem will be related to the radial Schrodinger equation for the Hydrogen atom.

1. Using dimensionalysis, what is the characteristic length scale for this problem?

$$\underbrace{-\frac{d^2\psi}{dx^2}}_{\frac{1}{L^2}} \underbrace{- \frac{2m\alpha}{\hbar^2 r}}_{\frac{2m\alpha}{\hbar^2 L}}\psi = \frac{2mE}{\hbar^2}\psi$$

$$\frac{1}{L^2} = \frac{2m\alpha}{\hbar^2 L} \longrightarrow \boxed{L = \frac{\hbar^2}{2m\alpha}}$$

2. Call this length scale L. Rewrite the Schrodinger equation in terms of a dimensionless variable $\xi = x/L$.

$$-L^2\frac{d^2\psi}{dx^2} - \frac{L^2 \cdot 2m\alpha}{\hbar^2 x}\psi = \frac{L^2\, 2mE}{\hbar^2}\psi$$

$$\rightarrow \boxed{-\frac{d^2\psi}{d\xi^2} - \frac{1}{\xi}\psi = \frac{E}{\hbar^2/2mL^2}\psi}$$

3. Using dimensionalysis, what is the characteristic energy scale for this problem?

$$\frac{\hbar^2}{2mL^2} = \frac{\hbar^2}{2m\left(\frac{\hbar^2}{2m\alpha}\right)^2} = \boxed{\frac{2m\alpha^2}{\hbar^2} = \frac{\alpha}{L}}$$