Name:

Quiz 5

Consider the two state system with states labeled by $|1\rangle$ and $|2\rangle$, and the Hamiltonian

$$H = E_0 \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} = E_0(i|1\rangle\langle 2| - i|2\rangle\langle 1|). \qquad (1)$$

1. What are the eigenvalues and eigenvectors of this Hamiltonian?

$$\det\begin{pmatrix} -\lambda & i \\ -i & -\lambda \end{pmatrix} \doteq 0 \rightarrow \lambda^2 - 1 = 0 \rightarrow \lambda = \pm 1$$

$$\lambda = 1: \begin{pmatrix} -1 & i \\ -i & -1 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 0 \rightarrow -c_1 + i c_2 = 0 \rightarrow |\psi_+\rangle \propto \begin{pmatrix} i \\ 1 \end{pmatrix}$$

$$\lambda = -1: \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 0 \rightarrow c_1 + i c_2 = 0 \rightarrow |\psi_-\rangle \propto \begin{pmatrix} -i \\ 1 \end{pmatrix}$$

Summary:

$E = +E_0$	$E = -E_0$
$\lvert\psi_+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix}$	$\lvert\psi_-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 1 \end{pmatrix}$

2. Express the state $(|1\rangle + |2\rangle)/\sqrt{2}$ in terms of the eigenvectors.

$$\frac{1}{\sqrt{2}}(|1\rangle + |2\rangle) = |\psi_+\rangle\langle\psi_+|\frac{1}{\sqrt{2}}(|1\rangle + |2\rangle) + |\psi_-\rangle\langle\psi_-|\frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$$

$$= \frac{-i+1}{2}|\psi_+\rangle + \frac{+i+1}{2}|\psi_-\rangle$$

(Your result may differ depending on the definition of the eigenstates.)

For example, if $|\psi_-\rangle = i\frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$, then

$$\frac{1}{\sqrt{2}}(|1\rangle + |2\rangle) = \frac{1}{1+i}(|\psi_+\rangle + |\psi_-\rangle).$$