

Name:

Quiz 6

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$H = \hbar\omega \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$|\psi(0^-)\rangle = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

1. If the state of the system is given by $|\psi(0^-)\rangle$ just before an A measurement is made at time $t = 0$, what are the possible outcomes of the A measurement and what are the probabilities of those outcomes?

eigenvalue	eigenvector	probability
+1	$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$	$ \frac{1}{\sqrt{2}} (1 \ 1 \ 0) \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} ^2 = \frac{2^2}{6} = \frac{2}{3}$
-1	$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$	0
0	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$	$ (0 \ 0 \ 1) \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} ^2 = \frac{1}{3}$

2. If many identical measurements were made, what would be the average value of A obtained?

$$\langle A \rangle = \frac{2}{3} \cdot 1 + 0 \cdot (-1) + \frac{1}{3} \cdot 0 = \frac{2}{3}$$

$$= \frac{1}{\sqrt{3}} (1 \ 1 \ 1) \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{3} (1 \ 1 \ 1) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \frac{2}{3} \checkmark$$

3. Suppose the measurement in part 1. yields a value of +1. What is the state of the system immediately after the measurement?

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

4. Following 3. what is the state of system for $t > 0$ with the H shown at the top of the page?

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{-i\omega t} + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{-2i\omega t}$$