Name:

Final Exam - PHY 4604 - Fall 2017
December 13, 2017
3:00P-5:00P, NPB 1002

Directions: Please clear your desk of everything except for pencils and pens. The exam is closed book, and you are not allowed calculators or formula sheets. Leave substantial space between you and your neighbor. Show your work on the space provided on the exam. I can provide additional scratch paper if needed.

Each exam question, (a),. . ., (d), is worth 5 points, and the entire exam is out of 100 points. Some formulas are given with the relevant question.

1. **Short answer:**

   (a) Write down the time dependent Schrodinger equation in one dimension.

   (b) For the one dimensional Schrodinger equation give an expression for the probability current.

   (c) What are the allowed values of l and for the $n = 3$ levels of the Hydrogen atom? What are the allowed values of m for each of the l values? Thus, how many $m = 1$ states are there for $n = 3$?

   (d) If two operators commute, what can you conclude about their eigenstates?

2. **One Dimensional Schrodinger Equation:**
Consider the potential $V(x) = V_o > 0$ for $-a < x < a$ and $V(x) = 0$ elsewhere. In the following take the energy to be larger than V_o: $E > V_o > 0$.

   (a) What is the general form of the solutions to the time independent Schrodinger equation for $-a < x < a$ and for $x > a$? Make sure to define all wave vectors in terms of the energy, E, and V_o.

   (b) Because $V(x) = V(-x)$, the solutions to the time independent Schrodinger equation may be taken to be even, $\psi(x) = \psi(-x)$, or odd, $\psi(x) = -\psi(-x)$. In the following look for an *even* solution. What is the general form of an even solution? For this solution, write down the boundary conditions at $x = a$.

(c) Match boundary conditions at $x = a$ and solve for the wave function of part (b) up to an overall multiplicative factor.

(d) If $\psi(x)$ is proportional to $\cos(kx + \varphi)$ for $x > a$, what is $\tan(\varphi)$? What happens to φ as $V_o \to 0$?

3. **Harmonic Oscillator:**
The Hamiltonian of the one-dimensional harmonic oscillator is

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2 x^2.$$

Suppose state of system at $t = 0$ is

$$\psi(x,0) = \alpha\psi_0(x) + \beta e^{i\varphi}\psi_2(x),$$

where $\psi_n(x)$ for $n = 0, 1, 2, \ldots$ are the energy eigenstates of the Hamiltonian. The constants α, β, and φ are real, and the wave function is normalized because $\alpha^2+\beta^2 = 1$.

(a) What is the wave function, $\psi(x,t)$, at time $t > 0$?

(b) Compute the expectation values of x and p at time t, where

$$\begin{aligned} x &= \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-) \\ p &= i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-). \end{aligned}$$

(c) Compute the expectation values of x^2 and p^2 at time t.

(d) Compute the uncertainty principle product $\Delta x \Delta p$. At what time(s) is this a minimum? What is the minimum value of $\Delta x \Delta p$ for this wave function in terms of α, β, φ? If you are allowed to vary α and β such that $\alpha^2 + \beta^2 = 1$, what do you think the minimum value of $\Delta x \Delta p$ will be and why?

4. **Formalism:**
The Pauli spin matrices are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(a) At $t = 0$ a measurement of $S_y = (\hbar/2)\sigma_y$ yields $\hbar/2$. What is the state of the system?

(b) After the measurement, the system evolves under the Hamiltonian, $H = \mu_B B \sigma_x$. What are the eigenstates and eigenvalues of this Hamiltonian?

(c) What is $|\psi(t)\rangle$ using the results from parts (a) and (b)?

(d) A measurement of S_z is made at time $t > 0$. What are the possible outcomes and the associated probabilities of this measurement? Interpret your result in terms of a what you know about a spin in a magnetic field.

5. **Angular momentum:**

(a) Compute the commutator $[(L_y)^2 + (L_z)^2, (L_x)^2]$.

(b) What is the maximum total angular momentum quantum number, j, that a spin 5/2 atom and a spin 3/2 atom ($j_1 = 5/2$ and $j_2 = 3/2$) can produce? Write down this state in terms of the product states: $|j_1, m_1\rangle \otimes |j_2, m_2\rangle$.

(c) Letting j be the maximum total angular momentum quantum number from (b), express the state $|j, j-1\rangle$ in terms of the product states.

(d) What is expectation value $\langle j, j-1|(J_x^2 + J_y^2)|j, j-1\rangle$ for general j, where J_x and J_y are the x and y components of the angular momentum operator, J?