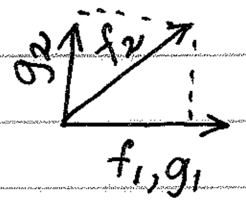
Gram-Schmidt orthogonalization:

Let $|f_1\rangle, |f_2\rangle, |f_3\rangle, \ldots$ be linearly independent, but not necessarily orthogonal.
We can use this procedure to get a new set of vectors, $|g_1\rangle, |g_2\rangle, \ldots$, which are orthogonal.

$$|g_1\rangle = |f_1\rangle$$



$$|g_2\rangle = |f_2\rangle - |g_1\rangle \frac{\langle g_1|f_2\rangle}{\langle g_1|g_1\rangle}$$

$$\rightarrow \langle g_1|g_2\rangle = 0$$

$$|g_3\rangle = |f_3\rangle - |g_1\rangle \frac{\langle g_1|f_3\rangle}{\langle g_1|g_1\rangle} - |g_2\rangle \frac{\langle g_2|f_3\rangle}{\langle g_2|g_2\rangle}$$

$$\rightarrow \quad 0 = \langle g_1|g_3\rangle = \langle g_2|g_3\rangle$$

$$\vdots$$