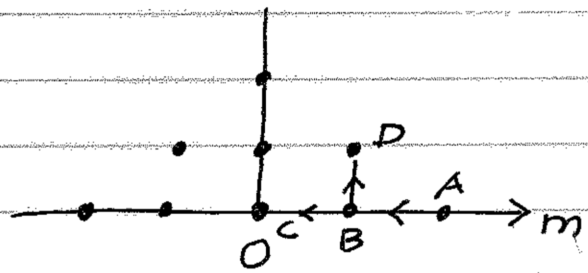
Clebsh-Gordon coefficients for 1×1:

m_1	m_2	m
1	1	2
1	0	1
1	-1	0
0	1	1
0	0	0
0	-1	-1
-1	1	0
-1	0	-1
-1	-1	-2



1 $j=0$ state
3 $j=1$ states
5 $j=2$ states

A. $|j=2, m=2\rangle = |1,1\rangle \otimes |1,1\rangle$

$$J_- |2,2\rangle = \hbar\sqrt{2(2+1)-2(2-1)}\, |2,1\rangle = \hbar 2 |2,1\rangle$$
$$= (J_-^{(1)} + J_-^{(2)}) |1,1\rangle \otimes |1,1\rangle$$
$$= \hbar\sqrt{2}\, |1,0\rangle \otimes |1,1\rangle + \hbar\sqrt{2}\, |1,1\rangle \otimes |1,0\rangle$$

B. $\rightarrow |2,1\rangle = \frac{1}{\sqrt{2}} \left(|1,0\rangle \otimes |1,1\rangle + |1,1\rangle \otimes |1,0\rangle \right)$

One can apply $J_- = J_-^{(1)} + J_-^{(2)}$ again to go from B: $|2,1\rangle$ to C: $|2,0\rangle$, etc.

D: $|1,1\rangle$ is orthogonal to $|2,1\rangle$.

$$\Rightarrow |1,1\rangle = \underbrace{\frac{-1}{\sqrt{2}}}_{\text{convention}} \left(|1,0\rangle \otimes |1,1\rangle - |1,1\rangle \otimes |1,0\rangle \right)$$

For the remainder of the lecture work on the CG coef. HW problem.