Solution

Name:

Quiz 1

In the following consider the wave function $\psi(x) = C\sqrt{2}$ for $0 \leq x \leq 1/3$, $\psi(x) = C$ for $1/3 \leq x \leq 1$ and $\psi(x) = 0$ otherwise.

1. What is the constant, C, so that the wave function is normalized?

$$1 = \int_0^1 dx\, |\psi(x)|^2 = 2C^2 \cdot \frac{1}{3} + C^2 \cdot \frac{2}{3} = \frac{4C^2}{3}$$

$$\rightarrow \boxed{C = \frac{\sqrt{3}}{2}}$$

2. What is the expectation value of x for this wave function?

$$\langle x \rangle = \int_0^{1/3} 2C^2 x\, dx + \int_{1/3}^1 C^2 x\, dx$$

$$= \frac{2C^2}{2} x^2 \Big|_0^{1/3} + \frac{C^2}{2} x^2 \Big|_{1/3}^1$$

$$= \underbrace{\frac{3}{4} \cdot \frac{1}{9}}_{1/12} + \underbrace{\frac{3}{8}\left(1 - \frac{1}{9}\right)}_{1/3} = \boxed{\frac{5}{12} = \langle x \rangle}$$

3. What is the expectation value of x^2 for this wave function?

$$\langle x^2 \rangle = \int_0^{1/3} 2C^2 x^2 dx + \int_{1/3}^1 C^2 x^2 dx$$

$$= \frac{3}{2} \cdot \frac{1}{3} x^3 \Big|_0^{1/3} + \frac{3}{4} \cdot \frac{1}{3} x^3 \Big|_{1/3}^1$$

$$= \frac{1}{2} \cdot \frac{1}{27} + \frac{1}{4} \cdot \frac{26}{27} = \frac{1}{2} \cdot \frac{14}{27} = \boxed{\frac{7}{27} = \langle x^2 \rangle}$$