Solution Name:

Quiz 2

For the harmonic oscillator hamiltonian, $H = \hbar\omega(a_+a_- + \frac{1}{2})$, the position operator may be written as

$$p = i\sqrt{\frac{\hbar m\omega}{2}}\,(a_+ - a_-)\,. \tag{1}$$

At $t = 0$ the wave function is given by

$$\psi(x,0) = \frac{1}{\sqrt{2}}\,(\psi_2(x) + i\psi_4(x))\,. \tag{2}$$

1. What is the wave function at time t: $\psi(x,t)$?

$$\psi(x,t) = \frac{1}{\sqrt{2}}\psi_2(x)e^{-i\frac{5}{2}\omega t} + \frac{i}{\sqrt{2}}\psi_4(x)e^{-i\frac{9}{2}\omega t}$$

2. What is the expectation value of p as a function of time?

0 since a_+ & a_- do not couple ψ_2 & ψ_4

3. What is the expectation value of p^2 as a function of time?

$$p^2 = -\frac{\hbar m\omega}{2}\left(\underbrace{a_+a_+}_{2\to4} + \underbrace{a_-a_-}_{4\to2} - \underbrace{a_+a_-}_{2\to2\ \&\ 4\to4} - \underbrace{a_-a_+}\right)$$

$$\langle p^2\rangle = \frac{1}{2}\frac{\hbar m\omega}{2}\int dx\left(\psi_2^* e^{i\frac{5}{2}\omega t} - i\,\psi_4^* e^{i\frac{9}{2}\omega t}\right)(\text{"}\quad\text{"})$$
$$\left(\psi_2 e^{-i\frac{5}{2}\omega t} + i\,\psi_4 e^{-i\frac{9}{2}\omega t}\right)$$

4. What is the expectation value of H as a function of time?

$$\langle p^2\rangle = \frac{\hbar m\omega}{4}\left((2\cdot2+1) + (2\cdot4+1) + i\sqrt{4}\sqrt{3}\,e^{-i2\omega t} - i\sqrt{4}\sqrt{3}\,e^{i2\omega t}\right)$$

$$= \frac{\hbar m\omega}{2}\left(7 + 2\sqrt{3}\sin(2\omega t)\right)$$

$$\langle E\rangle = \sum_n |C_n|^2 E_n = \frac{1}{2}\hbar\omega\left(2+\frac{1}{2}\right) + \frac{1}{2}\hbar\omega\left(4+\frac{1}{2}\right) = \hbar\omega\left(3+\frac{1}{2}\right)$$
$$= \frac{7}{2}\hbar\omega$$