Solution

Name:

Quiz 3

$$H = E_0 \begin{pmatrix} \frac{1}{2} & i\frac{\sqrt{3}}{2} \\ -i\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

1. What are the eigenvalues of the above Hamiltonian, H?

$$\det \begin{pmatrix} \frac{1}{2}-\lambda & \frac{i\sqrt{3}}{2} \\ \frac{-i\sqrt{3}}{2} & -\frac{1}{2}-\lambda \end{pmatrix} = 0 = -\frac{1}{4} + \lambda^2 - \frac{3}{4} \rightarrow \lambda^2 = 1 \rightarrow \lambda = \pm 1$$

and $\boxed{E = \pm E_0}$

2. What are the eignevectors of the Hamiltonian?

$$E_0 \begin{pmatrix} \frac{1}{2} & \frac{i\sqrt{3}}{2} \\ \frac{-i\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = E_0 \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \rightarrow E_0 \begin{pmatrix} -\frac{1}{2} & \frac{i\sqrt{3}}{2} \\ \frac{-i\sqrt{3}}{2} & -\frac{3}{2} \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = 0 \rightarrow \frac{1}{2} C_1 = \frac{i\sqrt{3}}{2} C_2$$

$$\boxed{E = E_0: \quad |\psi_+\rangle = \begin{pmatrix} \frac{i\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}}$$

$$E \begin{pmatrix} \frac{1}{2} & \frac{i\sqrt{3}}{2} \\ \frac{-i\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = -E_0 \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \rightarrow E_0 \begin{pmatrix} \frac{3}{2} & \frac{i\sqrt{3}}{2} \\ \frac{-i\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = 0 \rightarrow \frac{i\sqrt{3}}{2} C_1 = \frac{1}{2} C_2$$

$$\boxed{E = -E_0: \quad |\psi_-\rangle = \begin{pmatrix} \frac{1}{2} \\ \frac{i\sqrt{3}}{2} \end{pmatrix}}$$

3. For the eigenvector, $|\psi\rangle$, with the largest eigenvalue, what is $|\psi\rangle\langle\psi|$?

$$|\psi_+\rangle\langle\psi_+| = \begin{pmatrix} \frac{i\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}\begin{pmatrix} \frac{-i\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{i\sqrt{3}}{2} \cdot \frac{-i\sqrt{3}}{2} & \frac{i\sqrt{3}}{2} \cdot \frac{1}{2} \\ \frac{1}{2} \cdot \frac{-i\sqrt{3}}{2} & \frac{1}{2} \cdot \frac{1}{2} \end{pmatrix}$$

$$\boxed{|\psi_+\rangle\langle\psi_+| = \begin{pmatrix} \frac{3}{4} & \frac{i\sqrt{3}}{4} \\ \frac{-i\sqrt{3}}{4} & \frac{1}{4} \end{pmatrix}}$$