Name:

Quiz 4

$$H = \hbar\omega \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad ; \quad \text{eigenvalue: } +\hbar\omega;\ \text{eigenvector: } |E_+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{1}$$

$$\text{eigenvalue: } -\hbar\omega;\ \text{eigenvector: } |E_-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2}$$

$$S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad ; \quad \text{eigenvalue: } +\frac{\hbar}{2};\ \text{eigenvector: } |S_{y,+}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} \tag{3}$$

$$\text{eigenvalue: } -\frac{\hbar}{2};\ \text{eigenvector: } |S_{y,-}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix} \tag{4}$$

1. At $t = 0$ an S_y measurement is made with result $+\hbar/2$. What is the state of the system right after the measurement?

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

2. The system then evolves in time with the hamiltonian, H, above. What is the state of the system at time $t > 0$?

$$\boxed{|\psi(t)\rangle = e^{-iE_+t/\hbar}|E_+\rangle\langle E_+|\psi(0)\rangle + e^{-iE_-t/\hbar}|E_-\rangle\langle E_-|\psi(0)\rangle}$$

$$\langle E_+|\psi(0)\rangle = \frac{1}{\sqrt{2}}(1\ \ 1)\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} = \frac{1+i}{2}$$

$$\langle E_-|\psi(0)\rangle = \frac{1}{\sqrt{2}}(1\ \ -1)\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} = \frac{1-i}{2}$$

$$\boxed{|\psi(t)\rangle = e^{-i\omega t}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\left(\frac{1+i}{2}\right) + e^{i\omega t}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}\left(\frac{1-i}{2}\right)}$$

3. At time t a second measurement of S_y is made. What are the possible outcomes and associated probabilities?

outcome

$\boxed{+\frac{\hbar}{2}}$: $|\langle S_{y,+}|\psi(t)\rangle|^2 = \left|\frac{1}{\sqrt{2}}(1\ \ -i)|\psi(t)\rangle\right|^2$

$$= \left|\frac{1}{2}(1-i)\left(\frac{1+i}{2}\right)e^{-i\omega t} + \frac{1}{2}(1+i)\left(\frac{1-i}{2}\right)e^{i\omega t}\right|^2 = \left|\frac{e^{i\omega t}+e^{-i\omega t}}{2}\right|^2$$

$$\boxed{= \cos^2(\omega t)}$$

$\boxed{-\frac{\hbar}{2}}$: $|\langle S_{y,-}|\psi(t)\rangle|^2$

$$= \left|\frac{1}{\sqrt{2}}(-i\ \ 1)|\psi(t)\rangle\right|^2 = \left|\frac{1}{2}(1-i)\left(\frac{1+i}{2}\right)e^{-i\omega t} + \frac{1}{2}(-i-1)\left(\frac{1-i}{2}\right)e^{i\omega t}\right|^2$$

$$= \left|\frac{e^{-i\omega t}-e^{i\omega t}}{2}\right|^2 = |-i\sin\omega t|^2 = \boxed{\sin^2(\omega t)}$$